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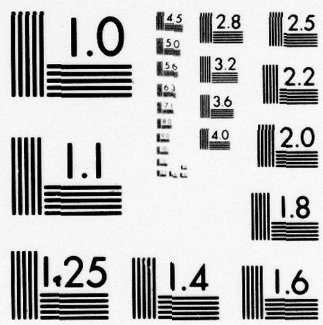

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OFF-CENTER, LOW VELOCITY, TRANSVERSE NORMAL IMPACT OF A SIMPLY SUPPORTED PLATE.

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A theoretical solution for the response of a simply supported plate to off-center low speed transverse impact is presented. This solution is empirically corrected, on the basis of viscoelastic beam analysis, to admit damping. Comparison of the numerical predictions of the corrected theory with central impact test results on graphite-epoxy composite laminates shows good agreement.

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The structural dynamic response model consists of a special orthotropic plate impacted by a rigid mass. Initial displacement and velocity distributions were assumed and the solution method utilized the finite Fourier sine transform and Laplace transforms with respect to both space and time.

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INTRODUCTION

In the present study, we are interested in investigating the ability of fiber reinforced composite material structures to resist handling and impact loading. Indeed, these problems represent an important element in assessing the suitability of such structures for long term service utilization. Accordingly, the present investigation is directed toward the development of a technique to predict the strains in plates which are induced by relatively low speed (0 to 30 m/s), hard object, transverse normal impact.

We have considered the problem of the low velocity impact of a beam in several of our previous papers, references (1) and (2). Specifically reference (1) showed that the adoption of a finite impact width considerably reduced the number of terms of the series needed in the numerical evaluation. Unfortunately the work of Chou and Flis, reference (3), indicated that the theoretical solution tended to overpredict the strain response by approximately 20 to 30 percent. Thus, in reference (2) the authors considered the introduction of a damping mechanism, i.e., a viscoelastic effect, to improve the correlation between theory and experiment. In view of the above comments, it is seen that all of our previous efforts have been confined to one dimensional problems. To remove this deficiency from our investigations we shall consider the analysis of a special orthotropic plate, i.e., we shall investigate a region of two dimensional extent.

Recalling the complexity of the damped beam problem, reference (3), i.e., the computations required approximately one hundred pages, we questioned the advisability of proceeding directly to the solution of a damped plate problem. In addition, it was our considered opinion that we, as yet, do not have a complete physical understanding of the damping mechanisms which occur in graphite-epoxy systems. Thus, since a sufficient experimental basis does not exist for the damping mechanism investigated in reference (2), it does not appear reasonable to expend the additional effort required to produce such a complex analysis. To correct for this inadequacy in the theoretical development, the time variation of the mode shapes would be corrected in accord with the results determined in the damped beam investigation, reference (2). Since the undamped theoretical solution tends to form an upper bound for the experimental data, the adoption of any damping mechanism will improve the correlation between theory and experiment. Therefore, in view of the previous discussion, it was decided that we would limit the extent of our theoretical investigation by omitting any consideration of damping from this portion of the task.

It was discovered that the theoretical solution, as initially formulated, exhibited poor numerical characteristics. Indeed, one of the subcases of the analysis was particularly subject to numerical instability. The reformulation of the equations expanded the computational work to approximately three hundred pages so that none of this latter work will be included in the present work. Thus, one should proceed with caution when attempting to work in this area. Indeed, it is the authors' opinion that duplication of the equations listed in this report should not proceed under any circumstances. Additional results which avoid the numerical instabilities are obtainable by regrouping terms and employing asymptotic expansions.

This report extends the analysis of composite structural impact damage by considering a body of two dimensional extent. We envision that the theoretical solution coincides with the following physical problem. A mass of small dimensions, possessing a given velocity, is approaching a plate which is initially at rest. During impact the mass transfers its momentum to the region of the plate with which it is in contact. This information is utilized to ascertain initial conditions for the vibrating system. The plate and impactor remain in contact, over a fixed rectangular area, during the remainder of the first quarter of the first cycle of the motion. However, the theoretical solution, since it considers no damping mechanism, must be empirically corrected, on the basis of damped beam theory, to account for the effects of damping. Thus, the solution presented in this report accounts for off-central impact, provides the exact eigenvalues and eigenfunctions for a simply supported plate carrying a distributed impactor mass, and it is empirically altered to account for material damping.

ANALYSIS

From Timoshenko and Woinowsky-Krieger, reference (4), we recall that the differential equation for the transverse deflection of a special orthotropic plate and the normal moment-curvature relations are given by,

$$D_x w_{xxxx} + 2H w_{xxyy} + D_y w_{yyyy} = q$$

$$M_x = -(D_x w_{xx} + D_1 w_{yy})$$

$$M_y = -(D_y w_{yy} + D_1 w_{xx}) \qquad (1)$$

where D_x, D_y = flexural rigidities in the x and y directions

D_1 = flexural rigidity associated with Poisson's effect

H = flexural rigidity associated with Poisson's effect and shear

M_x, M_y = bending moment per unit length in x and y directions

q = transverse load per unit area

w = transverse deflection of the plate

x,y = distances measured along the x and y coordinate axes.

Note that in the previous equation, subscripts are utilized to denote differentiation with respect to the variable so indicated. In accord with our previous work, references (1) and (2), it is assumed that the impacting mass remains in contact with the plate during the initial phase of its motion; thus, in view of equation (1) the equation of motion for a plate carrying a distributed mass may be written,

$$D_x w_{xxxx} + 2H w_{xxyy} + D_y w_{yyyy} \tag{2}$$
$$+ \left\{ \rho h + \frac{M}{4\varepsilon_1\varepsilon_2}\left[U(x-c+\varepsilon_1) - U(x-c-\varepsilon_1)\right]\left[U(y-d+\varepsilon_2) - U(y-d-\varepsilon_2)\right]\right\} w_t$$
$$= 0$$

where c,d = distance to impact point in x and y directions

h = thickness of plate

M = mass of impactor

t = time

U(x), unit step function = $\begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$\varepsilon_1, \varepsilon_2$ = half width of impact zone in x and y directions

ρ = mass density of plate material.

It is convenient to introduce the Laplace time transform, i.e.,

$$\bar{w}(x,y,p) = \int_0^\infty e^{-pt}\, w(x,y,t)\, dt \tag{3}$$

where p = time transform parameter

$\bar{w}$ = time transform of deflection.

Recalling from our original discussion that the plate is initially confined to the x-y plane, i.e., w (x,y,o) = 0, and noting our previous work, references (1) and (2), we assume ,

$$w_t(x,y,o) = V\left[U(x-c+\varepsilon_1) - U(x-c-\varepsilon_1)\right]\left[U(y-d+\varepsilon_2) - U(y-d-\varepsilon_2)\right] \tag{4}$$

where V = impact corrected velocity of impactor and plate.

Applying the Laplace transform, i.e., equation (3), to equation (2), utilizing our previous observation and equation (4), we obtain,

$$D_x \bar{w}_{xxxx} + 2H \bar{w}_{xxyy} + D_y \bar{w}_{yyyy} \tag{5}$$
$$+ p^2 \left\{ \rho h + \frac{M}{4\varepsilon_1\varepsilon_2}\left[U(x-c+\varepsilon_1) - U(x-c-\varepsilon_1)\right]\left[U(y-d+\varepsilon_2) - U(y-d-\varepsilon_2)\right]\right\} \bar{w}$$
$$= C_1\left[U(x-c+\varepsilon_1) - U(x-c-\varepsilon_1)\right]\left[U(y-d+\varepsilon_2) - U(y-d-\varepsilon_2)\right]$$

where $C_1 = \left(\rho h + \frac{M}{4\varepsilon_1\varepsilon_2}\right) V$

Noting that the functions $[U(x-c+\varepsilon_1) - U(x-c-\varepsilon_1)]$ and $[U(y-d+\varepsilon_2) - U(y-d-\varepsilon_2)]$ effectively limit the contribution of $\bar{w}$ to the neighborhood of the impact point (c,d) in accord with our previous work, references (1) and (2), we shall assume that,

$$[U(x-c+\varepsilon_1)-U(x-c-\varepsilon_1)][U(y-d+\varepsilon_2)-U(y-d-\varepsilon_2)]\bar{w}$$
$$\cong \bar{w}(c,d,p)[U(x-c+\varepsilon_1)-U(x-c-\varepsilon_1)][U(y-d+\varepsilon_2)-U(y-d-\varepsilon_2)]$$

Introducing the previous approximation into equation (5), we find,

$$D_x\bar{w}_{xxxx}+2H\bar{w}_{xxyy}+D_y\bar{w}_{yyyy}+\rho h p^2\bar{w} \tag{6}$$
$$=[C_1-C_2p^2\bar{w}(c,d,p)][U(x-c+\varepsilon_1)-U(x-c-\varepsilon_1)][U(y-d+\varepsilon_2)-U(y-d-\varepsilon_2)]$$

where $C_2=\dfrac{M}{4\varepsilon_1\varepsilon_2}$

Next we define the finite-sine transform by,

$$\tilde{f}(x,m,t)=\int_0^b f(x,y,t)\sin\frac{m\pi y}{b}\,dy \tag{7}$$

where $\tilde{f}(x,m,t)$ = finite-sine transform of the arbitrary function f.

m = finite-sine transform parameter

Further, we recall that for the problem of a simply supported plate the boundary conditions along the edges y = o and y = b, i.e., equation (1), may be written,

$$w(x,0,t)=w(x,b,t)=0$$
$$M_y(x,0,t)=M_y(x,b,t)=0$$

However, since we have adopted a sine solution, i.e., equation (7), it may be shown that the deflection function satisfies the following conditions,

$$w(x,0,t)=w(x,b,t)=0$$
$$w_{xx}(x,0,t)=w_{yy}(x,0,t)=0$$
$$w_{xx}(x,b,t)=w_{yy}(x,b,t)=0$$

These latter results imply, in view of equation (1), that $M_y(x,o,t) = M_y(x,b,t) = 0$ and therefore, a sine expansion of our function satisfies the boundary conditions imposed along the lines y = o and y = b. Thus, it is admissible to employ the finite-sine transform, i.e., equation (7), on the equation of motion, i.e., equation (6), to obtain,

$$\tilde{\bar{w}}_{xxxx} - 2\frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2 \tilde{\bar{w}}_{xx} + \left[\frac{\rho h}{D_x}p^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]\tilde{\bar{w}}$$

$$= C_m\left[C_3 - C_4 p^2 \bar{w}(c,d,p)\right]\left[U(x-c+\epsilon_1) - U(x-c-\epsilon_1)\right] \tag{8}$$

where $C_3 = \left(\frac{\rho h}{D_x} + \frac{M}{4\epsilon_1\epsilon_2 D_x}\right)V$

$$C_4 = \frac{M}{4\epsilon_1\epsilon_2 D_x}$$

$$C_m = \frac{2b}{\pi m}\sin\frac{\pi m\epsilon_2}{b}\sin\frac{\pi m d}{b}$$

Finally, we introduce the Laplace space transform by,

$$\hat{w}(s,y,t) = \int_0^{\infty} e^{-sx} w(x,y,t)\,dx \tag{9}$$

where s = space transform parameter

$\hat{w}(s,y,t)$ = space transform of transverse deflection.

Utilizing the Laplace space transform, i.e., equation (9), on the basic system equation, i.e., equation (8), we discover that,

$$s^4\hat{\tilde{\bar{w}}} - s^3\tilde{\bar{w}}(0,m,p) - s^2\tilde{\bar{w}}_x(0,m,p) - s\tilde{\bar{w}}_{xx}(0,m,p) - \tilde{\bar{w}}_{xxx}(0,m,p)$$

$$-2\frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2\left[s^2\hat{\tilde{\bar{w}}} - s\tilde{\bar{w}}(0,m,p) - \tilde{\bar{w}}_x(0,m,p)\right]$$

$$+\left[\frac{\rho h}{D_x}p^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]\hat{\tilde{\bar{w}}}$$

$$= C_m\left[C_3 - C_4p^2\bar{w}(c,d,p)\right]\cdot\frac{1}{s}e^{-cs}\left(e^{\epsilon_1 s} - e^{-\epsilon_1 s}\right) \tag{10}$$

However, it may be shown that the boundary conditions along the edges x = o and x = a, i.e., $w(o,y,t) = M_x(o,y,t) = w(a,y,t) = M_x(a,y,t) = 0$, are equivalent to the conditions,

$$\tilde{\bar{w}}(o,m,p) = \tilde{\bar{w}}(a,m,p) = 0$$

$$\tilde{\bar{w}}_{xx}(o,m,p) = \tilde{\bar{w}}_{xx}(a,m,p) = 0 \tag{11}$$

Applying this set of boundary conditions to the equation of motion, i.e., equation (10), and performing some algebraic reduction, we have,

$$\hat{\tilde{\tilde{w}}} = \frac{1}{a-\bar{a}}\Big\{\Big[\frac{a}{s^2-\bar{a}} - \frac{\bar{a}}{s^2-a}\Big]\tilde{\tilde{w}}_x(0,m,p) + \Big[\frac{1}{s^2-a} - \frac{1}{s^2-\bar{a}}\Big]\tilde{\tilde{w}}_{xxx}(0,m,p) + C_m\big[C_3 - C_4 p^2 \bar{w}(c,d,p)\big]\cdot \frac{1}{s}\Big[\frac{1}{s^2-a} - \frac{1}{s^2-\bar{a}}\Big]e^{-cs}\big(e^{\epsilon_1 s} - e^{-\epsilon_1 s}\big)\Big) \quad (12)$$

where $a = \frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2 + i\sqrt{\frac{\rho h}{D_x}p^2 + \frac{D_x D_y - H^2}{D_x^2}\left(\frac{\pi m}{b}\right)^4}$

$$\bar{a} = \frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2 - i\sqrt{\frac{\rho h}{D_x}p^2 + \frac{D_x D_y - H^2}{D_x^2}\left(\frac{\pi m}{b}\right)^4}$$

Inverting the equation of motion, i.e., equation (12), with respect to x, we obtain,

$$\begin{aligned}\tilde{\tilde{w}} = {} & \frac{1}{2b_r b_i(b_r^2+b_i^2)}\cdot \\ & \cdot\Big\{\big[b_r(-b_r^2+3b_i^2)\cosh b_r x \sin b_i x + b_i(3b_r^2-b_i^2)\sinh b_r x \cos b_i x\big]\tilde{\tilde{w}}_x(0,m,p) \\ & + \big[b_r \cosh b_r x \sin b_i x - b_i \sinh b_r x \cos b_i x\big]\tilde{\tilde{w}}_{xxx}(0,m,p) \\ & + \frac{C_m}{b_r^2+b_i^2}\big[C_3 - C_4 p^2 \bar{w}(c,d,p)\big]\cdot \\ & \cdot\Big\langle\Big[(b_r^2-b_i^2)\sinh b_r(x-c+\epsilon_1)\sin b_i(x-c+\epsilon_1) \\ & \quad + 2b_r b_i\{1-\cosh b_r(x-c+\epsilon_1)\cos b_i(x-c+\epsilon_1)\}\Big]U(x-c+\epsilon_1) \\ & - \Big[(b_r^2-b_i^2)\sinh b_r(x-c-\epsilon_1)\sin b_i(x-c-\epsilon_1) \\ & \quad + 2b_r b_i\{1-\cosh b_r(x-c-\epsilon_1)\cos b_i(x-c-\epsilon_1)\}\Big]U(x-c-\epsilon_1)\Big\rangle\Big\} \quad (13)\end{aligned}$$

where $b_r = \left\{\frac{1}{2}\left[\frac{\rho h}{D_x}p^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{1/2} + \frac{1}{2}\frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2\right\}^{1/2}$

$$b_i = \left\{\frac{1}{2}\left[\frac{\rho h}{D_x}p^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{1/2} - \frac{1}{2}\frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2\right\}^{1/2}$$

In accord with our previous discussion, the solution represented by equation (13) must satisfy the boundary conditions along the edge x = a expressed in equation (11). Differentiating equation (13) twice with respect to x and utilizing the results in the boundary conditions along edge x = a, it is possible to determine expressions for $\tilde{\tilde{w}}_x(o,m,p)$ and $\tilde{w}_{xxx}(o,m,p)$. Substituting the results of these calculations into equation (13), we conclude that,

$$\tilde{\tilde{W}} = \frac{i}{4b_r^2 b_i^2 (b_r^2+b_i^2)^2} \left[C_3 - C_4 p^2 \bar{W}(c,d,p)\right] \cdot$$

$$\cdot \left\{ -\frac{1}{\sinh^2 b_r a + \sin^2 b_i a} \cdot \right.$$

$$\cdot \Big\langle b_r \left[\mathcal{A}_0 (-b_r^2 + 3b_i^2) + \mathcal{A}_1 (b_r^2+b_i^2) \right] \cosh b_r x \sin b_i x$$

$$+ b_i \left[\mathcal{A}_0 (3b_r^2 - b_i^2) - \mathcal{A}_1 (b_r^2+b_i^2) \right] \sinh b_r x \cos b_i x \Big\rangle$$

$$+ 2b_r b_i \Big\langle \left[(b_r^2 - b_i^2) \sinh b_r (x-c+\epsilon_1) \sin b_i (x-c+\epsilon_1) \right.$$

$$\left. + 2b_r b_i \left\{ 1 - \cosh b_r (x-c+\epsilon_1) \cos b_i (x-c+\epsilon_1) \right\} \right] U(x-c+\epsilon_1)$$

$$- \left[(b_r^2 - b_i^2) \sinh b_r (x-c-\epsilon_1) \sin b_i (x-c-\epsilon_1) \right.$$

$$\left. \left. + 2b_r b_i \left\{ 1 - \cosh b_r (x-c-\epsilon_1) \cos b_i (x-c-\epsilon_1) \right\} \right] U(x-c-\epsilon_1) \Big\rangle \right\} \quad (14)$$

where $\mathcal{A}_s = \sinh b_r (a-c+\epsilon_1) \sin b_i (a-c+\epsilon_1) - \sinh b_r (a-c-\epsilon_1) \sin b_i (a-c-\epsilon_1)$

$$\mathcal{A}_c = \cosh b_r (a-c+\epsilon_1) \cos b_i (a-c+\epsilon_1) - \cosh b_r (a-c-\epsilon_1) \cos b_i (a-c-\epsilon_1)$$

$$\mathcal{A}_0 = (b_r \mathcal{A}_s - b_i \mathcal{A}_c) \sinh b_r a \cos b_i a - (b_i \mathcal{A}_s + b_r \mathcal{A}_c) \cosh b_r a \sin b_i a$$

$$\mathcal{A}_1 = (b_r \mathcal{A}_s + b_i \mathcal{A}_c) \sinh b_r a \cos b_i a + (b_i \mathcal{A}_s - b_r \mathcal{A}_c) \cosh b_r a \sin b_i a$$

Inverting equation (14) with respect to y, we note that,

$$\bar{W} = \frac{1}{\pi} \left[C_3 - C_4 p^2 \bar{W}(c,d,p) \right] \cdot$$

$$\sum_{m=1}^{\infty} \frac{1}{m\, b_r^2 b_i^2 (b_r^2+b_i^2)^2} \sin \frac{\pi m \epsilon_2}{b} \sin \frac{\pi m d}{b} \sin \frac{\pi m y}{b} \cdot$$

$$\cdot \left\{ -\frac{1}{\sinh^2 b_r a + \sin^2 b_i a} \cdot \right.$$

$$\cdot \Big\langle b_r \left[(-b_r^2 + 3b_i^2) \mathcal{A}_0 + (b_r^2 + b_i^2) \mathcal{A}_1 \right] \cosh b_r x \sin b_i x$$

$$+b_i\left[(3b_r^2-b_i^2)\Omega_0-(b_r^2+b_i^2)\Omega_1\right]\sinh b_r x\cos b_i x\Big\rangle$$

$$+2b_rb_i\Big\langle\Big[(b_r^2-b_i^2)\sinh b_r(x-c+\epsilon_1)\sin b_i(x-c+\epsilon_1)$$

$$+2b_rb_i\{1-\cosh b_r(x-c+\epsilon_1)\cos b_i(x-c+\epsilon_1)\}\Big]U(x-c+\epsilon_1)$$

$$-\Big[(b_r^2-b_i^2)\sinh b_r(x-c-\epsilon_1)\sin b_i(x-c-\epsilon_1)$$

$$+2b_rb_i\{1-\cosh b_r(x-c-\epsilon_1)\cos b_i(x-c-\epsilon_1)\}\Big]\cdot \tag{15}$$

$$\cdot U(x-c-\epsilon_1)\Big\rangle\Big\}$$

To continue our solution, we evaluate equation (15) at the impact point, i.e., x = c and y = d, and solve the resulting expression for $\overline{w}(c,d,p)$. Introducing the result of this computation into equation (15) we finally obtain the time transform of the transverse displacement, or,

$$\overline{w}(x,y,p)=\frac{1}{\pi}\cdot\frac{C_3}{1+\frac{C_4}{\pi}p^2\sum_{m=1}^{\infty}\phi_m}\cdot\sum_{m=1}^{\infty}\xi_m \tag{16}$$

where $\xi_m=\dfrac{1}{m\,b_r^2b_i^2(b_r^2+b_i^2)^2}\sin\dfrac{\pi m\epsilon_2}{b}\sin\dfrac{\pi m d}{b}\sin\dfrac{\pi m y}{b}\cdot$

$$\cdot\Big\{-\frac{1}{\sinh^2 b_r a+\sin^2 b_i a}\cdot$$

$$\cdot\Big\langle b_r\left[(-b_r^2+3b_i^2)\Omega_0+(b_r^2+b_i^2)\Omega_1\right]\cosh b_r x\sin b_i x$$

$$+b_i\left[(3b_r^2-b_i^2)\Omega_0-(b_r^2+b_i^2)\Omega_1\right]\sinh b_r x\cos b_i x\Big\rangle$$

$$+2b_rb_i\Big\langle\Big[(b_r^2-b_i^2)\sinh b_r(x-c+\epsilon_1)\sin b_i(x-c+\epsilon_1)$$

$$+2b_rb_i\{1-\cosh b_r(x-c+\epsilon_1)\cos b_i(x-c+\epsilon_1)\}\Big]U(x-c+\epsilon_1)$$

$$-\Big[(b_r^2-b_i^2)\sinh b_r(x-c-\epsilon_1)\sin b_i(x-c-\epsilon_1)$$

$$+2b_rb_i\{1-\cosh b_r(x-c-\epsilon_1)\cos b_i(x-c-\epsilon_1)\}\Big]U(x-c-\epsilon_1)\Big\rangle\Big\}$$

$$\lambda_m = \frac{1}{m b_r^2 b_i^2 (b_r^2 + b_i^2)^2} \sin\frac{\pi m \epsilon_2}{b} \sin^2\frac{\pi m d}{b}$$

$$\mu_m = -\frac{1}{\sinh^2 b_r a + \sin^2 b_i a} \cdot$$

$$\cdot \Big\{ b_r \left[(-b_r^2 + 3b_i^2)\Omega_0 + (b_r^2 + b_i^2)\Omega_1 \right] \cosh b_r c \sin b_i c$$

$$+ b_i \left[(3b_r^2 - b_i^2)\Omega_0 - (b_r^2 + b_i^2)\Omega_1 \right] \sinh b_r c \cos b_i c \Big\}$$

$$\nu_m = 2 b_r b_i \Big\{ (b_r^2 - b_i^2) \sinh b_r \epsilon_1 \sin b_i \epsilon_1$$

$$+ 2 b_r b_i (1 - \cosh b_r \epsilon_1 \cos b_i \epsilon_1) \Big\}$$

$$\phi_m = \lambda_m (\mu_m + \nu_m)$$

To complete this investigation we must invert equation (16), i.e., the Laplace transform of the transverse deflection, and obtain the time response of the system. In reality we are concerned with the curvatures of the plate and not the displacement; however, since the technique to ascertain either is identical, we shall consolidate our considerations and only discuss the simplest of these quantities. To accomplish the above stated purpose we must utilize the complex inversion theorem for Laplace transforms. However, an inspection of equation (16) indicates that it is necessary to extend our discussion beyond the time transform parameter p and include the variables b_r and b_i, where the relations connecting b_r and b_i with p are recorded at the end of equation (13). To assist in this process the correspondence between the usual indented path of integration in the p-plane and the corresponding paths in the b_r and b_i - planes are indicated in figures 1 and 2. It is of particular importance to observe that figure 1 declares that the path of integration in the b_r-plane is dependent upon the flexural rigidities of the plate. Therefore, in view of the different paths indicated in these two figures it is necessary to subdivide the general solution into the following three cases:

1) both b_r and b_i are imaginary.

2) b_r is real and b_i is imaginary.

3) $b_r = \alpha + i\beta$ and $b_i = \beta + i\alpha$

where $2\alpha = \left\{ \left\langle \frac{\rho h}{D_x} p_y^2 + \frac{H^2 - D_x D_y}{D_x^2} \left(\frac{\pi m}{b}\right)^4 \right\rangle^{1/2} + \frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2 \right\}^{1/2}$

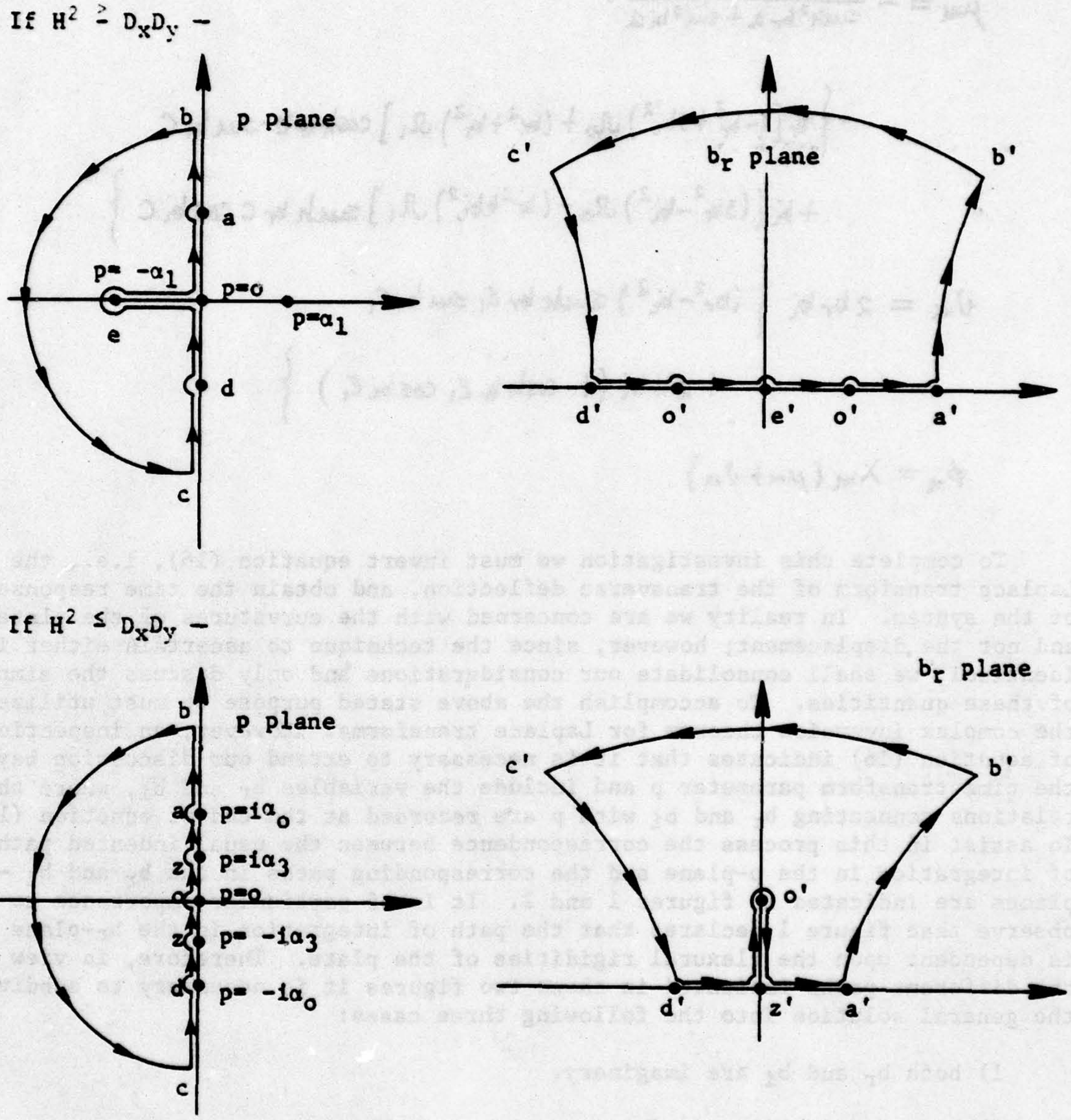

Figure 1 - Mapping from the P Plane to the BR Plane

Note: $\alpha_0 = \sqrt{\dfrac{D_y}{\rho h}}\left(\dfrac{\pi m}{b}\right)^2$; $\alpha_1 = \sqrt{\dfrac{H^2 - D_xD_y}{\rho h\, D_x}}\left(\dfrac{\pi m}{b}\right)^2$; $\alpha_3 = \sqrt{\dfrac{D_xD_y - H^2}{\rho h\, D_x}}\left(\dfrac{\pi m}{b}\right)^2$

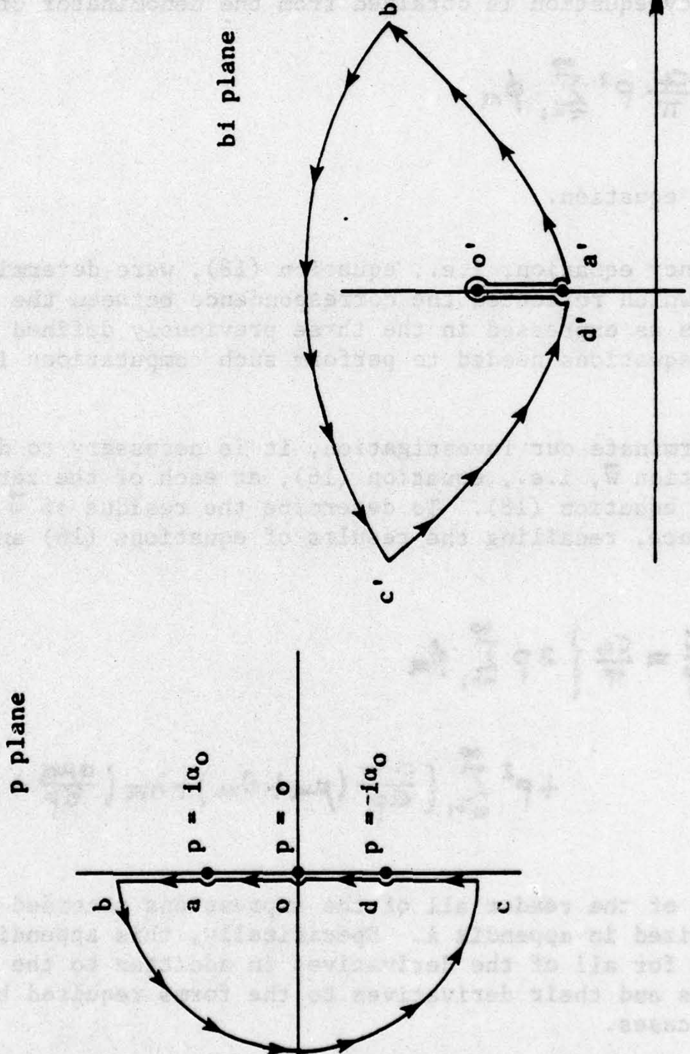

Figure 2 - Mapping from the P Plane to the BI Plane

$$2p = \left\{ \left\langle \frac{\rho h}{D_x} p_y^2 + \frac{H^2 - D_x D_y}{D_x^2} \left(\frac{\pi m}{b}\right)^4 \right\rangle^{1/2} - \frac{H}{D_x}\left(\frac{\pi m}{b}\right)^2 \right\}^{1/2} \tag{17}$$

Next, we require the location of all of the singularities which lie within and on the contours depicted in figures 1 and 2. Since the present analysis neglects damping, the singularities should lie along the y axis of the p-plane, thus, we need only consider points on the contours depicted on the various diagrams. The frequency equation is obtained from the denominator of equation (16), i.e.,

$$\chi = 1 + \frac{c_4}{\pi} p^2 \sum_{m=1}^{\infty} \phi_m \tag{18}$$

where χ = frequency equation.

Zeros of the frequency equation, i.e., equation (18), were determined by writing a computer program which reflected the correspondence between the p-plane and the b_r and b_i-planes as expressed in the three previously defined cases. A summary of all of the equations needed to perform such computations is included in appendix A.

In order to terminate our investigation, it is necessary to determine the residue of the function $\bar{\bar{w}}$, i.e., equation (16), at each of the zeros of the frequency equation, equation (18). To determine the residue of $\bar{\bar{w}}$ requires the derivative of χ, hence, recalling the results of equations (16) and (18), we find,

$$\chi' \equiv \frac{d\chi}{dp} = \frac{c_4}{\pi} \left\{ 2p \sum_{m=1}^{\infty} \phi_m + p^2 \sum_{m=1}^{\infty} \left[\frac{d\lambda_m}{dp} (\mu_m + \nu_m) + \lambda_m \left(\frac{d\mu_m}{dp} + \frac{d\nu_m}{dp} \right) \right] \right\} \tag{19}$$

For the convenience of the reader all of the expressions recorded in the above equation are summarized in appendix A. Specifically, this appendix includes the general expressions for all of the derivatives in addition to the transformation of all the functions and their derivatives to the forms required by the three previously defined cases.

Finally, utilizing the complex inversion integral for Laplace transforms and suitably correcting for the transformation between the p and b_r, b_i variables, we obtain,

$$w(x,y,t) = 2 \frac{c_2}{c_4} \sum_{p_y} \frac{\sum_{m=1}^{\infty} \xi_m}{\psi} \sin p_y t \tag{20}$$

where p_y = imaginary part of p

$$\psi = 2p_y \sum_{m=1}^{\infty} \phi_m - p_y^2 \sum_{m=1}^{\infty} \left[\left(\frac{d\lambda_m}{dp}\right)^* (\mu_m + \nu_m) + \lambda_m \left\langle \left(\frac{d\mu_m}{dp}\right)^* + \left(\frac{d\nu_m}{dp}\right)^* \right\rangle \right]$$

Note that although all of the equations between equation (16) and equation (19) are complex expressions, equation (20) is presented in real form, i.e., in the form in which it is employed in the numerical processes. Values for the real functions

$$\left(\frac{d\lambda m}{dp}\right)^*, \left(\frac{d\mu m}{dp}\right)^*, \text{ etc.}$$

may be found in appendix A corresponding to the three different cases.

Although equation (20) represents our solution of the plate impact problem, in view of our previous work (reference (2)), we recognize its insufficiencies because of the neglect of damping. Since the present work required approximately three hundred pages of computation it is unlikely that the extension to include damping will be attempted. However, since equation (20) appears to act as an upper bound solution, we shall modify it by the inclusion of an exponential factor which decreases with time in an attempt to include the effect of damping. Thus, we shall modify equation (20), in accord with the results recorded in reference (2), to obtain,

$$w(x,y,t) = 2\frac{C_3}{C_4} \sum_{p_y} \frac{\sum_{m=1}^{\infty} \xi_m}{\psi} e^{-\frac{1}{2}k p_y^2 t} \sin p_y t \tag{21}$$

where k = constant reflecting the equivalent damping of the vibrating system.

The solution represented by equation (21) and all of its similarly corrected second partial derivatives was programmed for numerical evaluation on a digital computer. However, it is important to record that the equations presented in this report are not numerically stable and considerable additional effect is required to construct numerically stable equivalences. In particular, case 2 is subject to considerable difficulty so that one should contact either of the authors before attempting to employ any of the equations listed in this report. Computational data were obtained from the experimental results presented by P.C. Chou and W. J. Flis (reference (3)). The results of the experimental tests and the theoretical calculations are presented in tables I through III and figures 3 through 17.

It is of general interest to observe that the correlation between the exponentially corrected solutions and the experimental results are reasonably

good. Indeed, most of the empirical information lies between or near the solutions corresponding to the two selected values of damping indicated in the figures. Only in the case of plate series B7 is there an obvious disagreement between the two sets of results. Fortunately, Chou (reference (3)) has published the photographic record of the strain response for this particular investigation and it may be seen from these results that the experimental data exhibits a behavior that is incompatible with the theoretical characteristics of our model. Thus, it would appear that an agency of unknown, but dissipative, origin was operative during this particular set of tests.

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TABLE I - Experimental and Theoretical Values of the Strain/Unit Velocity for B Plate Series

	Impactor Mass, kg	Velocity, M/Sec	$\varepsilon_{xx}/V \times 10^{-3}$, Sec/M Exp.	ε_{xx}/V K, Damping Constant 0	25×10^{-6}	100×10^{-6}	$\varepsilon_{yy}/V \times 10^{-3}$, Sec/M Exp.	ε_{yy}/V K, Damping Constant 0	25×10^{-6}	100×10^{-6}
B1	0.228	1.73	1.01	1.53	1.01	0.88	1.50	2.49	1.54	1.34
		2.45	1.10				1.62			
		2.99	1.24				1.79			
	0.453	1.73	1.58	2.01	1.49	1.37	2.33	3.22	2.28	2.10
		2.45	1.54				2.18			
		2.99	1.74				2.34			
	0.907	1.73	2.57	2.72	2.19	2.06	3.71	4.32	3.37	3.16
B2	0.228	1.73	0.90	1.23	1.02	0.87	1.11	1.69	1.37	1.12
		2.45	0.90				1.31			
		2.99	1.01				1.34			
	0.453	1.73	1.57	1.85	1.51	1.36	2.04	2.52	2.04	1.78
		2.45	1.52				1.96			
		2.99	1.54				2.21			
	0.907	1.73	2.23	2.66	2.26	2.11	3.19	3.66	3.01	2.78
B3	0.228	1.73	0.72	1.08	0.79	0.60	1.33	2.01	1.47	1.19
		2.45	0.76				1.45			
		2.99	0.85				1.52			
	0.453	1.73	1.07	1.38	1.07	0.96	2.08	2.58	2.05	1.87
		2.45	1.19				2.15			
		2.99	1.24				2.24			
	0.907	1.73	1.65	1.81	1.58	1.46	3.07	3.43	3.04	2.84
		2.45	1.72				3.27			
B4	0.228	1.73	0.59	1.11	0.75	0.52	0.96	1.81	1.19	0.79
		2.45	0.62				0.89			
		2.99	0.67				0.96			

TABLE I - Experimental and Theoretical Values of the Strain/Unit Velocity for B Plate Series (Continued)

	Impactor Mass, kg	Velocity, M/Sec	$\varepsilon_{xx}/V \times 10^{-3}$, Sec/M				$\varepsilon_{yy}/V \times 10^{-3}$, Sec/M			
			Exp.	K, Damping Constant			Exp.	K, Damping Constant		
				0	25×10^{-6}	100×10^{-6}		0	25×10^{-6}	100×10^{-6}
B4	0.453	1.73	1.03	1.39	1.06	0.82	1.42	2.23	1.68	1.25
		2.45	1.02				1.44			
		2.99	1.00				1.48			
	0.907	1.73	1.49	1.94	1.47	1.25	2.23	3.09	2.29	1.90
B5	0.228	1.73	0.99	1.30	0.94	0.85	1.34	1.66	1.10	0.99
		2.45	1.08				1.33			
		2.99	1.08				1.32			
	0.453	1.73	1.76	1.79	1.54	1.39	2.10	2.37	1.93	1.70
		2.45	1.79				2.15			
		2.99	1.76				1.98			
	0.907	1.73	2.50	2.48	2.23	2.09	3.12	3.22	2.80	2.58
B6	0.228	1.73	0.68	0.80	0.65	0.52	1.23	1.50	1.31	1.12
		2.45	0.65				1.22			
		2.99	0.70				1.31			
	0.453	1.73	1.16	1.20	1.06	0.88	2.08	2.29	2.11	1.85
		2.45	1.21				2.06			
		2.99	1.20				1.94			
	0.907	1.73	1.89	1.57	1.42	1.31	2.92	3.04	2.87	2.76
B7	0.228	1.73	0.43	0.68	0.51	0.45	0.78	1.35	1.17	1.05
		2.45	0.41				0.72			
		2.99	0.42				0.70			
	0.453	1.73	0.58	1.23	1.02	0.86	1.10	2.39	2.10	1.86
		2.45	0.65				1.13			
		2.99	0.67				1.15			
	0.907	1.73	0.98	1.70	1.51	1.36	1.71	3.37	3.11	2.87
		2.45	0.98				1.74			

TABLE II - Experimental and Theoretical Values of the Strain/Unit Velocity for F Plate Series

	Impactor Mass, kg	Velocity, M/Sec	$\varepsilon_{xx}/V \times 10^{-3}$, Sec/M				$\varepsilon_{yy}/V \times 10^{-3}$, Sec/M			
			Exp.	K, Damping Constant			Exp.	K, Damping Constant		
				0	25×10^{-6}	100×10^{-6}		0	25×10^{-6}	100×10^{-6}
F1	0.228	1.73	0.47	1.10	0.58	0.42	0.67	1.77	0.85	0.62
		2.45	0.48				0.79			
		2.99	0.54				0.87			
	0.453	1.73	0.74	1.52	0.93	0.72	1.18	2.42	1.38	1.08
		2.45	0.82				1.20			
		2.99	0.87				1.23			
	0.907	1.73	1.16	1.98	1.41	1.17	1.74	3.13	2.11	1.76
		2.45	1.21				1.75			
F2	0.228	1.73	0.49	0.82	0.49	0.40	0.67	1.08	0.59	0.48
		2.45	0.49				0.69			
		2.99	0.48				0.69			
	0.453	1.73	0.64	1.20	0.91	0.72	1.01	1.63	1.17	0.91
		2.45	0.69				1.22			
		2.99	0.70				1.01			
	0.907	1.73	1.18	1.88	1.39	1.17	1.69	2.52	1.82	1.50
		2.45	1.21				1.62			
F3	0.228	1.73	0.46	0.83	0.39	0.27	0.70	1.47	0.75	0.55
		2.45	0.48				0.70			
		2.99	0.47				0.73			
	0.453	1.73	0.58	1.05	0.66	0.50	1.04	1.91	1.25	0.99
		2.45	0.65				1.23			
		2.99	0.69				1.24			
	0.907	1.73	1.04	1.31	0.95	0.84	1.82	2.43	1.84	1.62
		2.45	1.06				1.79			

TABLE II - Experimental and Theoretical Values of the Strain/Unit Velocity for F Plate Series (Continued)

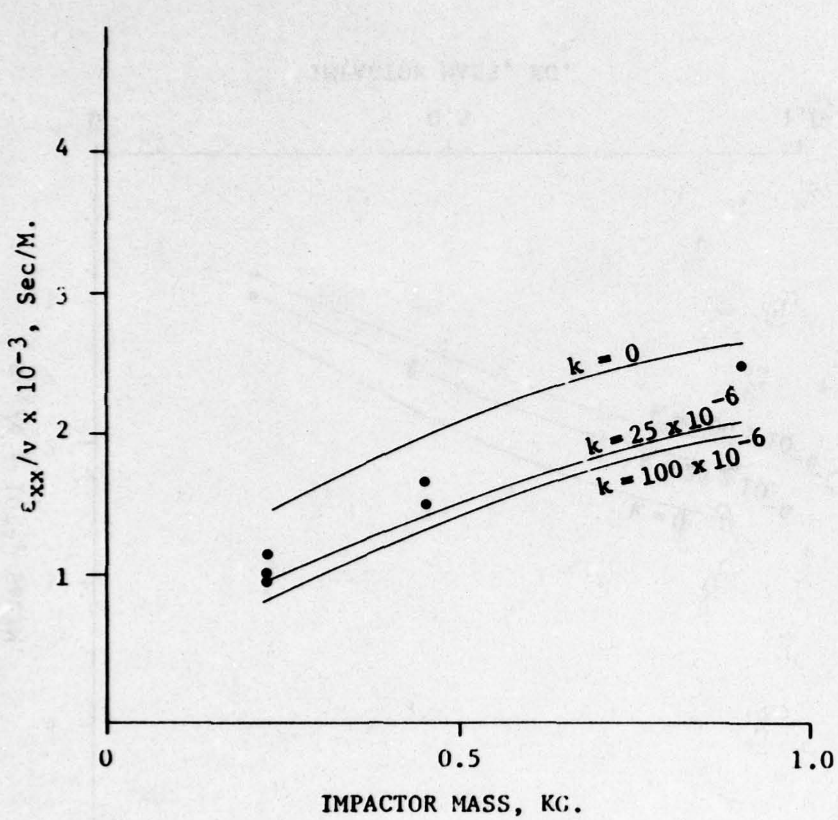
	Impactor Mass, Kg	Velocity, M/Sec	$\varepsilon_{xx}/V \times 10^{-3}$, Sec/M Exp.	$\varepsilon_{xx}/V \times 10^{-3}$, Sec/M K, Damping Constant 0	25×10^{-6}	100×10^{-6}	$\varepsilon_{yy}/V \times 10^{-3}$, Sec/M Exp.	$\varepsilon_{yy}/V \times 10^{-3}$, Sec/M K, Damping Constant 0	25×10^{-6}	100×10^{-6}
F4	0.228	1.73	0.51	0.77	0.38	0.23	0.66	1.26	0.61	0.37
		2.45	0.50				0.67			
		2.99	0.51				0.66			
	0.453	1.73	0.59	1.10	0.60	0.42	0.81	1.76	0.91	0.60
		2.45	0.61				0.77			
		2.99	0.57				0.79			
	0.907	1.73	0.93	1.38	0.90	0.71	1.24	2.18	1.38	1.05
		2.45	0.98				1.35			

TABLE III - Experimental and Theoretical Values of the Strain/Unit Velocity for H Plate Series

	Impactor Mass, kg	Velocity, M/Sec	$\varepsilon_{xx}/V \times 10^{-3}$, Sec/M Exp.	ε_{xx} K, Damping Constant 0	ε_{xx} K, Damping Constant 25×10^{-6}	ε_{xx} K, Damping Constant 100×10^{-6}	$\varepsilon_{yy}/V \times 10^{-3}$, Sec/M Exp.	ε_{yy} K, Damping Constant 0	ε_{yy} K, Damping Constant 25×10^{-6}	ε_{yy} K, Damping Constant 100×10^{-6}
H1	0.228	1.73	0.57	1.41	0.74	0.56	0.47	1.36	0.72	0.54
		2.45	0.60				0.49			
		2.99	0.68				0.55			
	0.453	1.73	1.05	1.92	1.18	0.95	0.91	1.86	1.15	0.92
		2.45	1.10				0.96			
		2.99	1.11				0.92			
	0.907	1.73	1.62	2.48	1.78	1.52	1.39	2.41	1.74	1.48
		2.45	1.62				1.42			
H2	0.228	1.73	0.59	1.23	0.70	0.53	0.55	1.03	0.54	0.39
		2.45	0.63				0.57			
		2.99	0.64				0.59			
	0.453	1.73	0.75	1.63	1.16	0.93	0.67	1.35	0.91	0.71
		2.45	0.94				0.78			
		2.99	1.04				0.87			
	0.907	1.73	1.40	2.18	1.68	1.51	1.10	1.85	1.29	1.16
H3	0.228	1.73	0.58	1.01	0.54	0.41	0.56	1.08	0.64	0.51
		2.45	0.59				0.56			
		2.99	0.60				0.55			
	0.453	1.73	0.70	1.36	0.95	0.74	0.75	1.47	1.11	0.89
		2.45	0.78				0.86			
		2.99	0.79				0.87			
	0.907	1.73	1.26	1.91	1.40	1.22	1.39	2.10	1.63	1.44
		2.45	1.34				1.47			

TABLE III - Experimental and Theoretical Values of the Strain/Unit Velocity for H Plate Series (Continued)

	Impactor Mass, kg	Velocity, M/Sec	$\varepsilon_{xx}/V \times 10^{-3}$,Sec/M				$\varepsilon_{yy}/V \times 10^{-3}$,Sec/M			
			Exp.	K, Damping Constant			Exp.	K, Damping Constant		
				0	25×10^{-6}	100×10^{-6}		0	25×10^{-6}	100×10^{-6}
H4	0.228	1.73	0.65	1.09	0.51	0.33	0.54	1.01	0.46	0.31
		2.45	0.65				0.54			
		2.99	0.66				0.53			
	0.453	1.73	0.80	1.48	0.81	0.55	0.71	1.38	0.77	0.52
		2.45	0.84				0.73			
		2.99	0.87				0.76			
	0.907	1.73	1.37	1.77	1.17	0.92	1.19	1.67	1.11	0.88
		2.45	1.31				1.04			



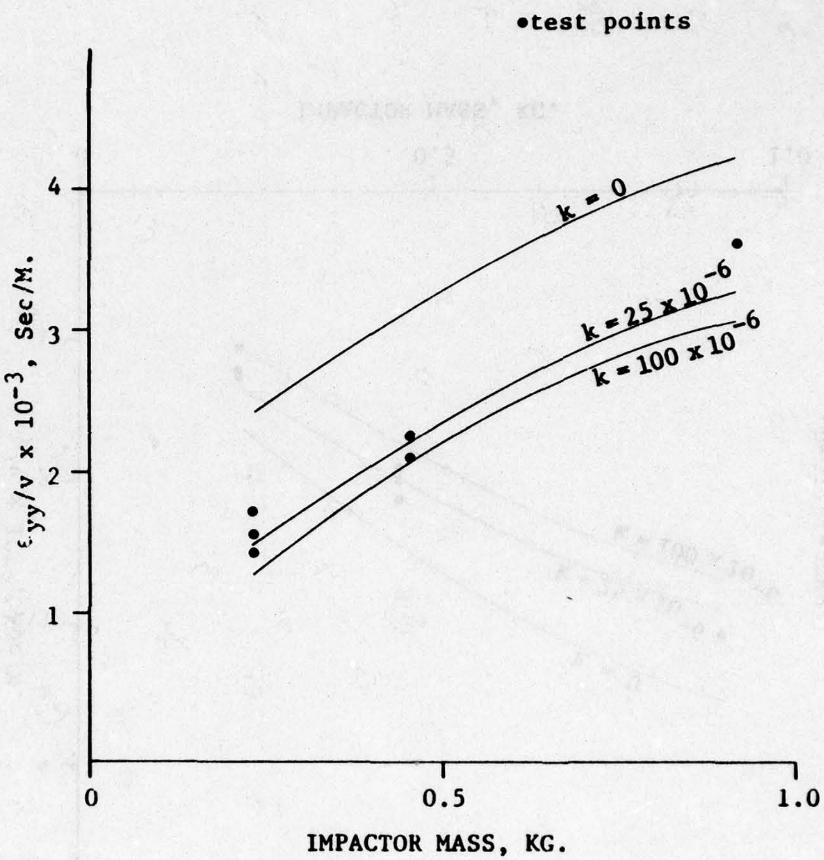


Figure 3 - Comparison Between Theoretical and Experimental Results for Plate Series B1

Note: Physical configuration and parametric values for each of the plate series may be found in reference (3).

* ϵ_{xx}/v represents the strain in the x-direction per unit velocity

** ϵ_{yy}/v represents the strain in the y-direction per unit velocity

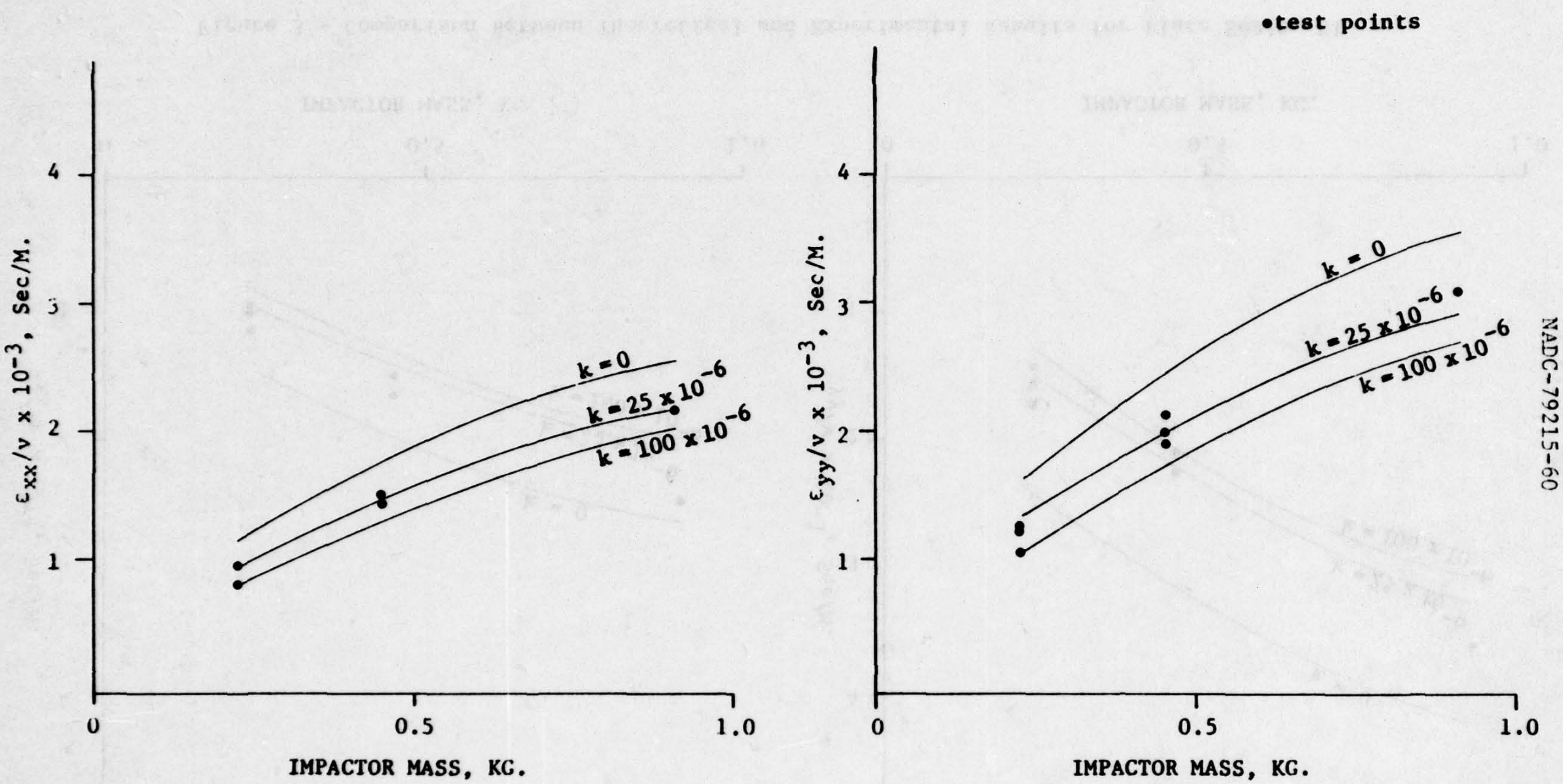


Figure 4 - Comparison Between Theoretical and Experimental Results for Plate Series B2

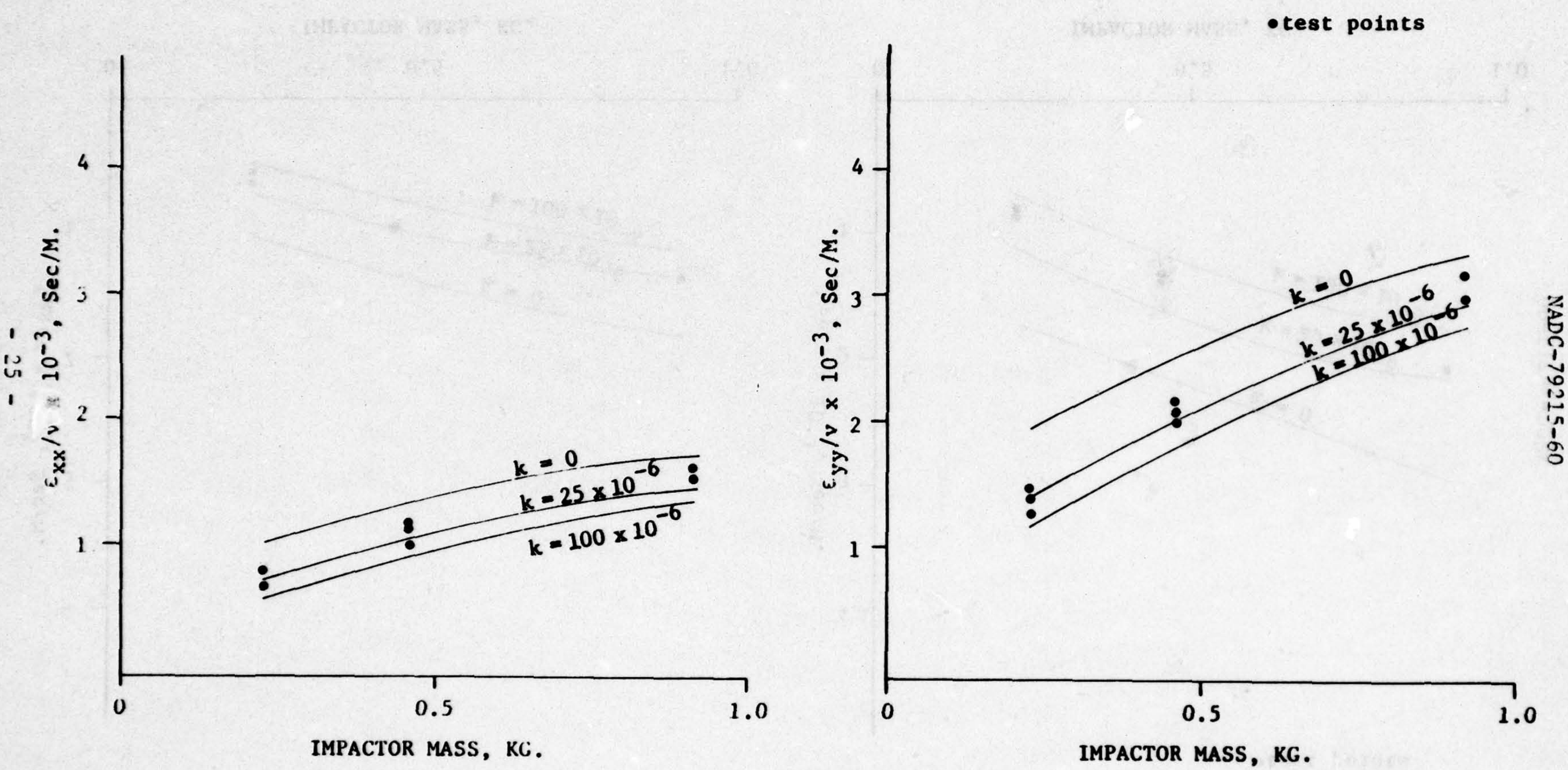


Figure 5 - Comparison Between Theoretical and Experimental Results for Plate Series B3

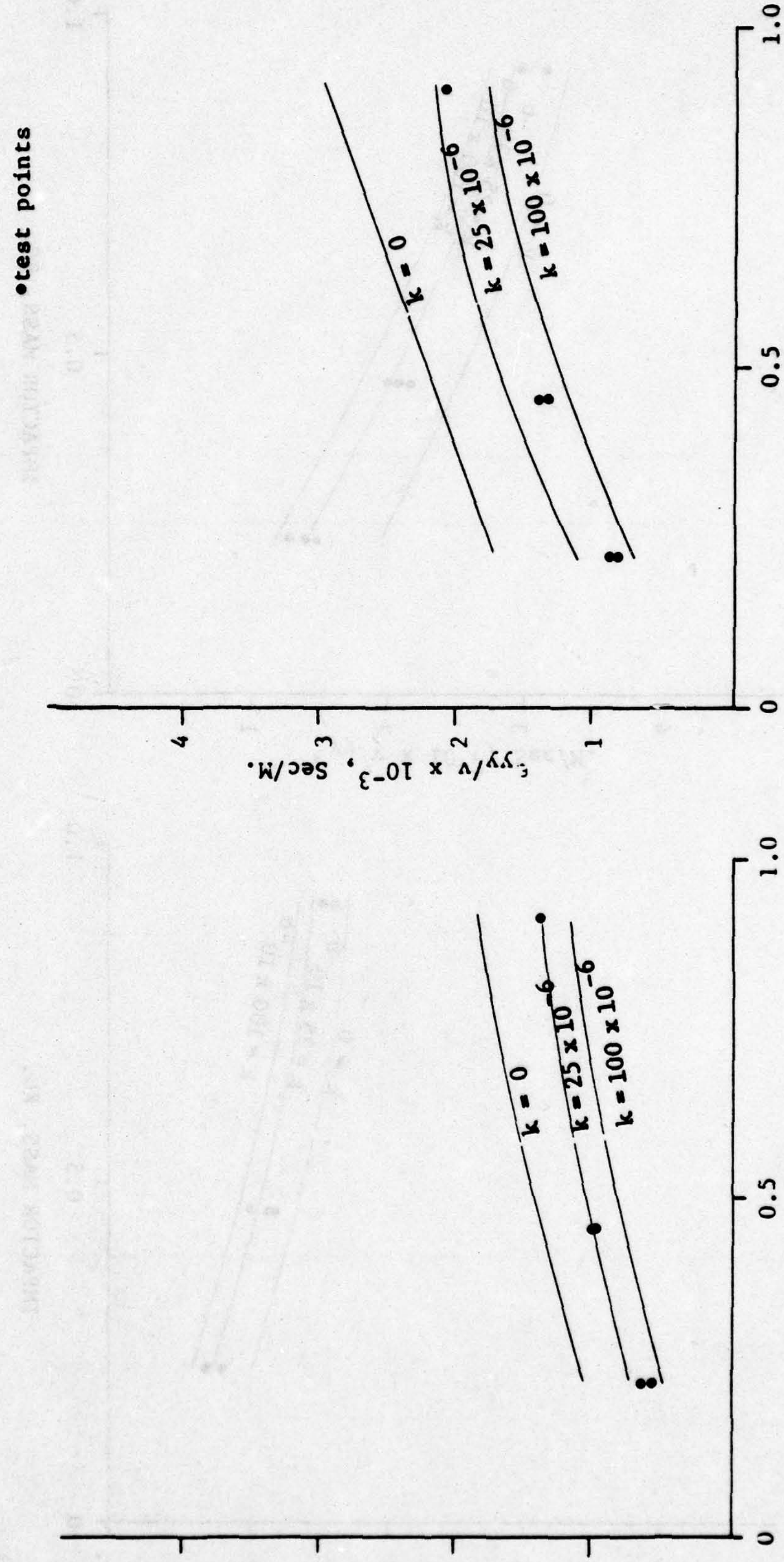


Figure 6 - Comparison Between Theoretical and Experimental Results for Plate Series B4

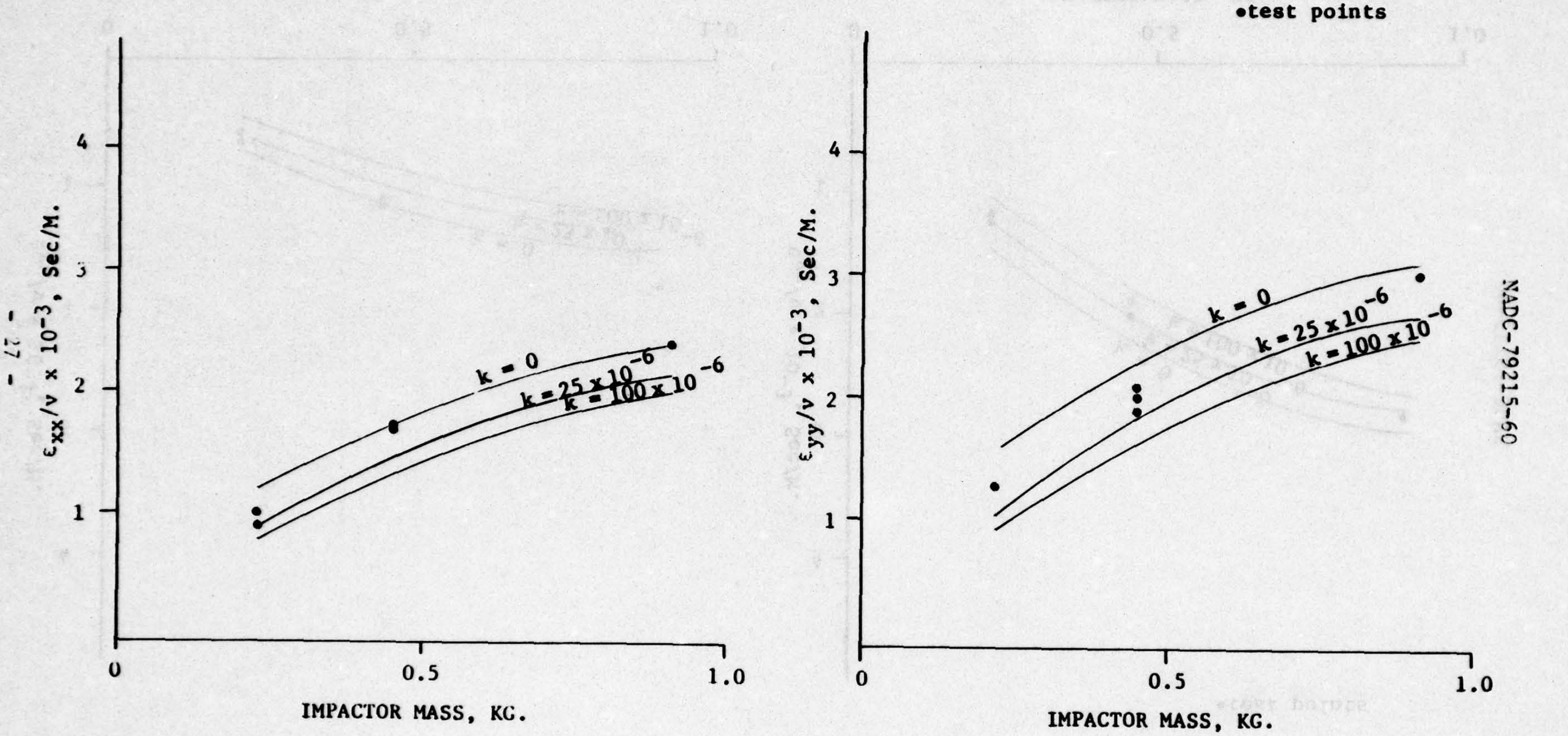


Figure 7 - Comparison Between Theoretical and Experimental Results for Plate Series B5

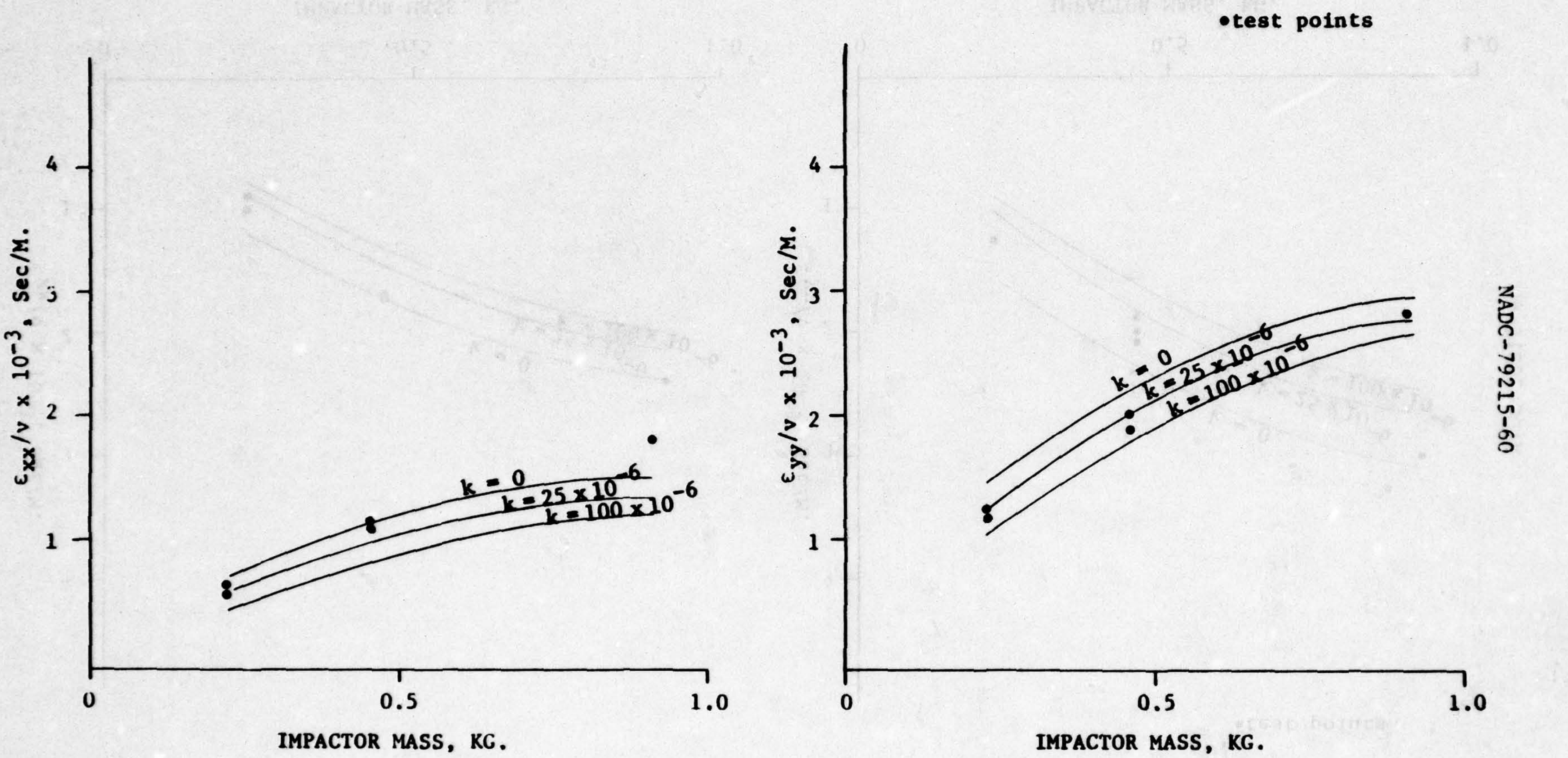


Figure 8 - Comparison Between Theoretical and Experimental Results for Plate Series B6

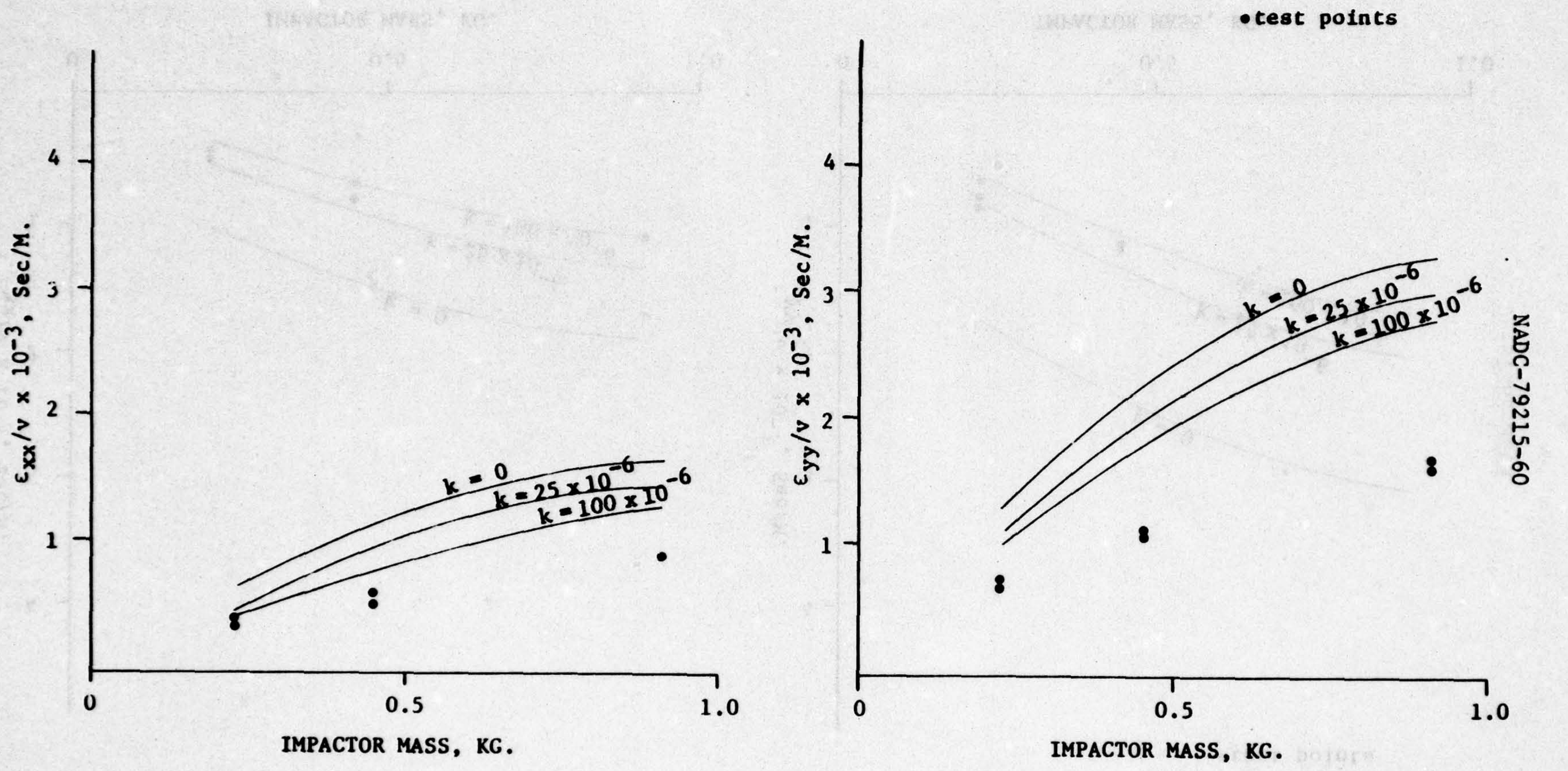


Figure 9 - Comparison Between Theoretical and Experimental Results for Plate Series B7

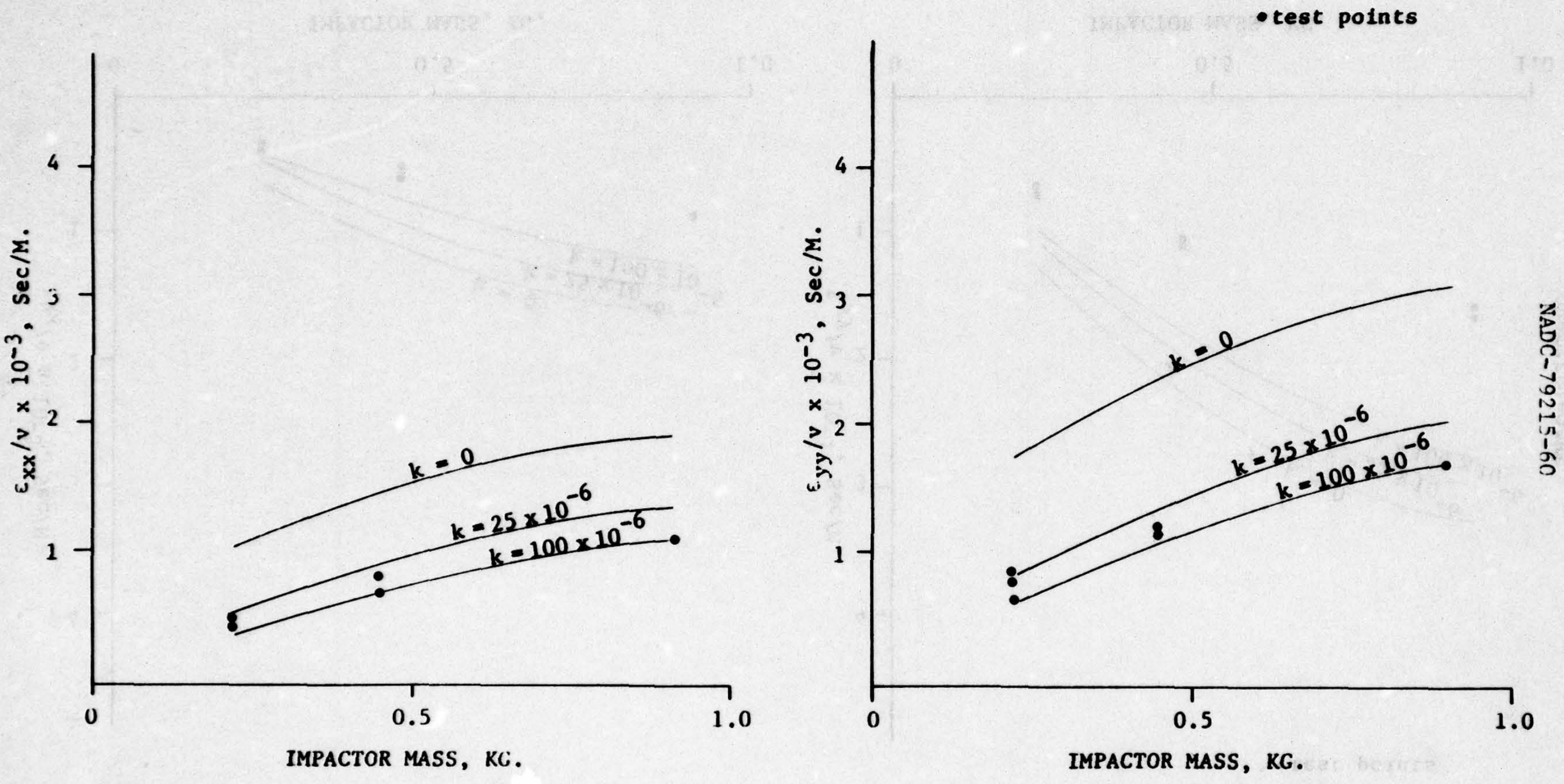


Figure 10 - Comparison Between Theoretical and Experimental Results for Plate Series F1

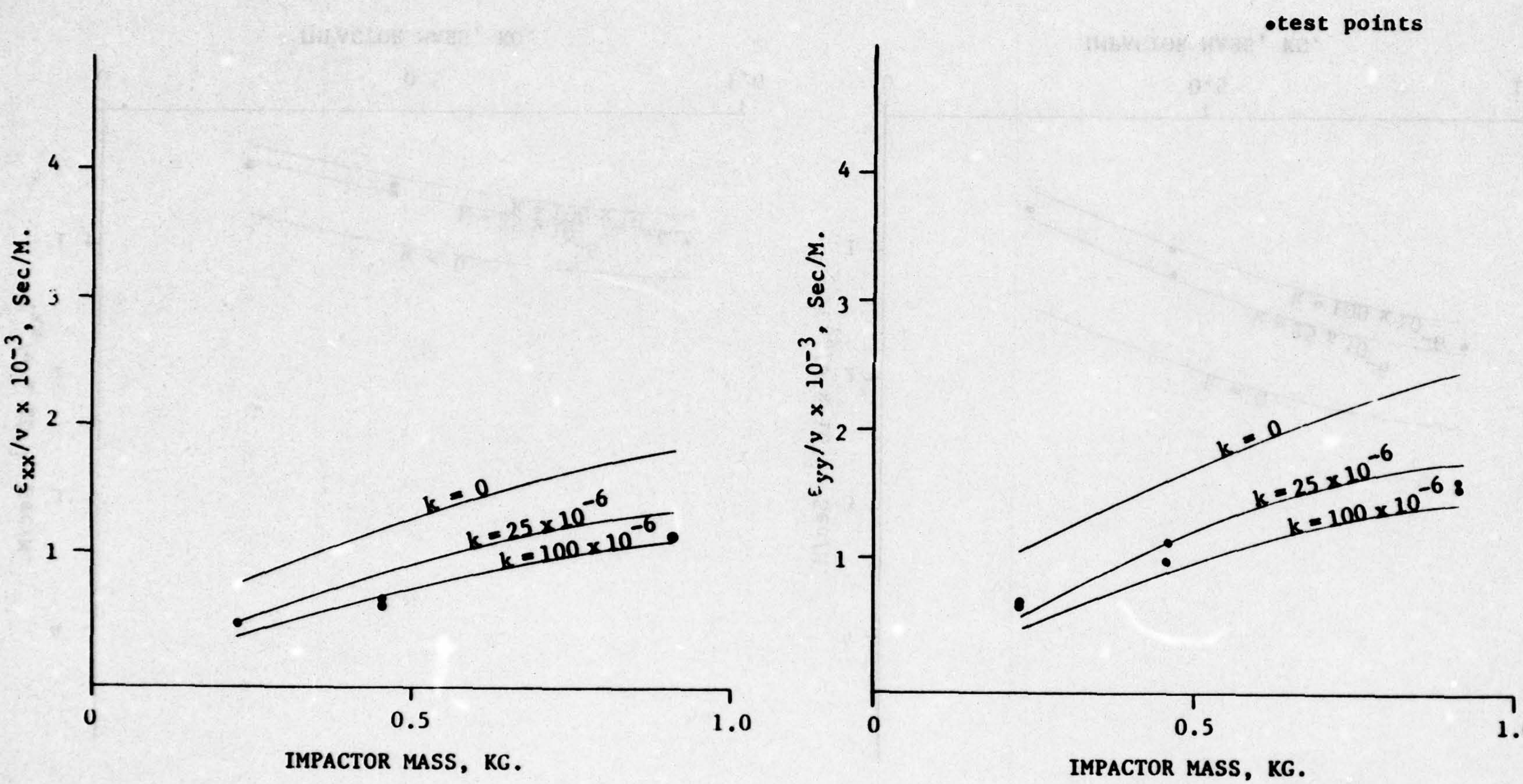


Figure 11 - Comparison Between Theoretical and Experimental Results for Plate Series F2

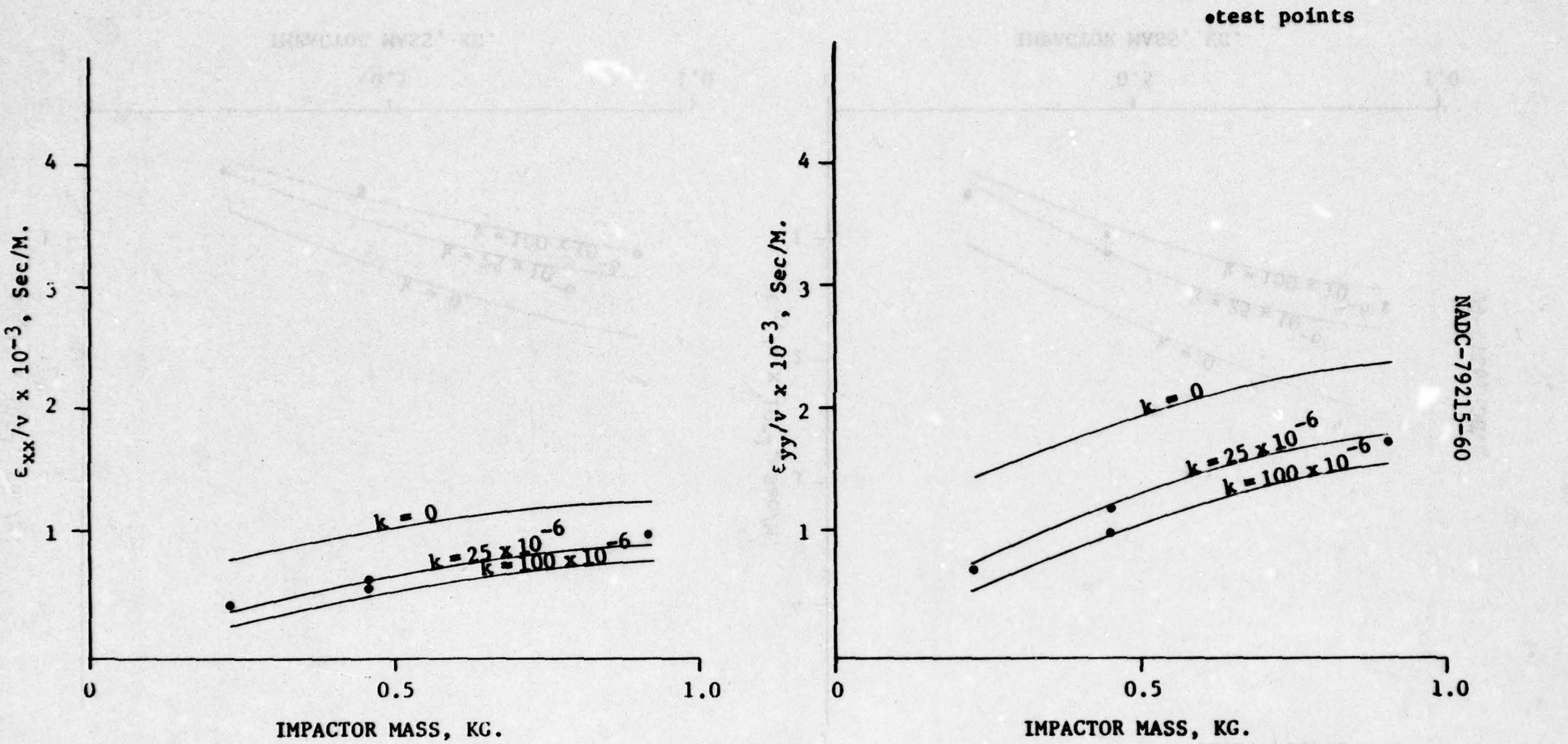


Figure 12 - Comparison Between Theoretical and Experimental Results for Plate Series F3

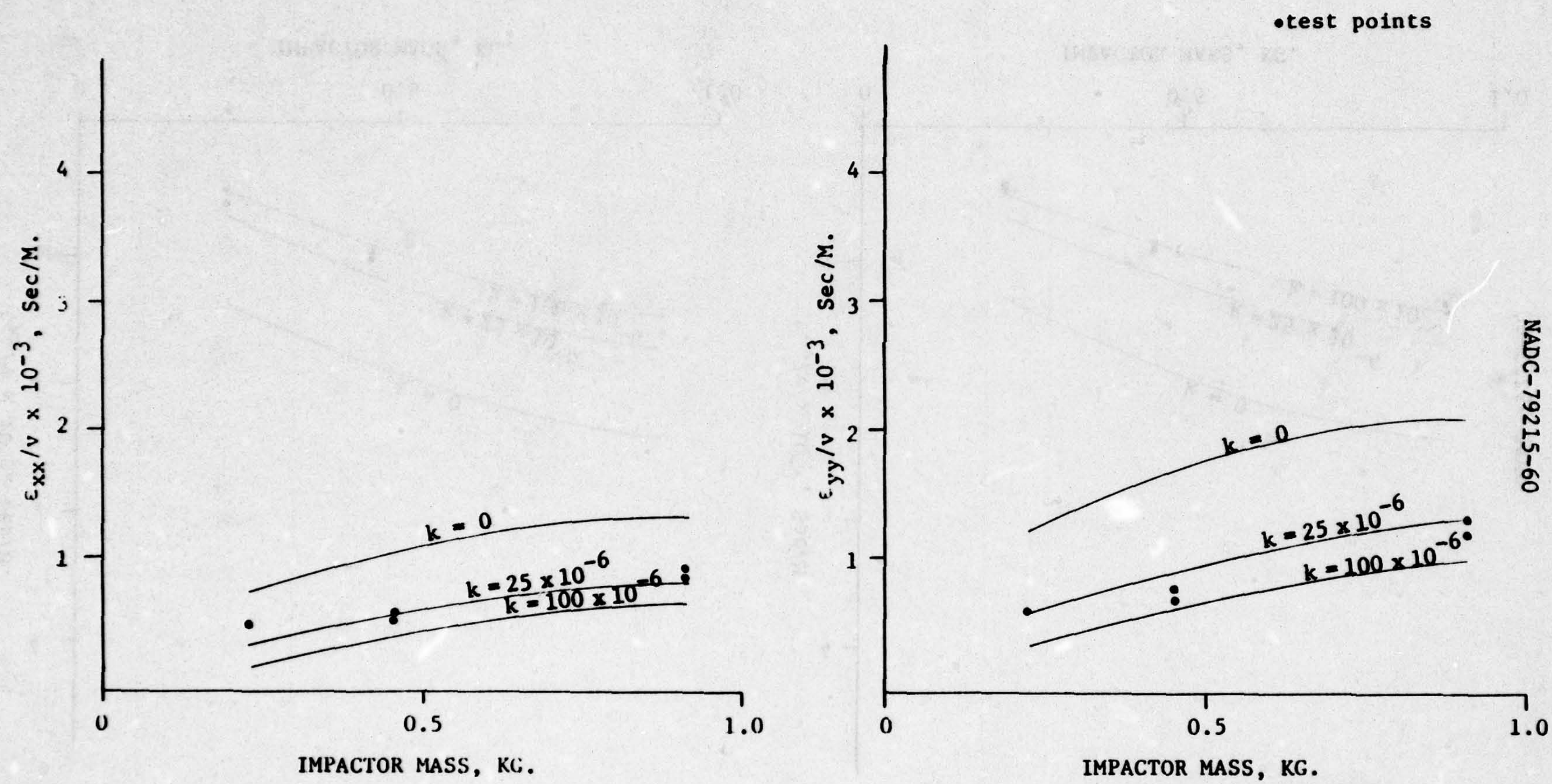


Figure 13 - Comparison Between Theoretical and Experimental Results for Plate Series F4

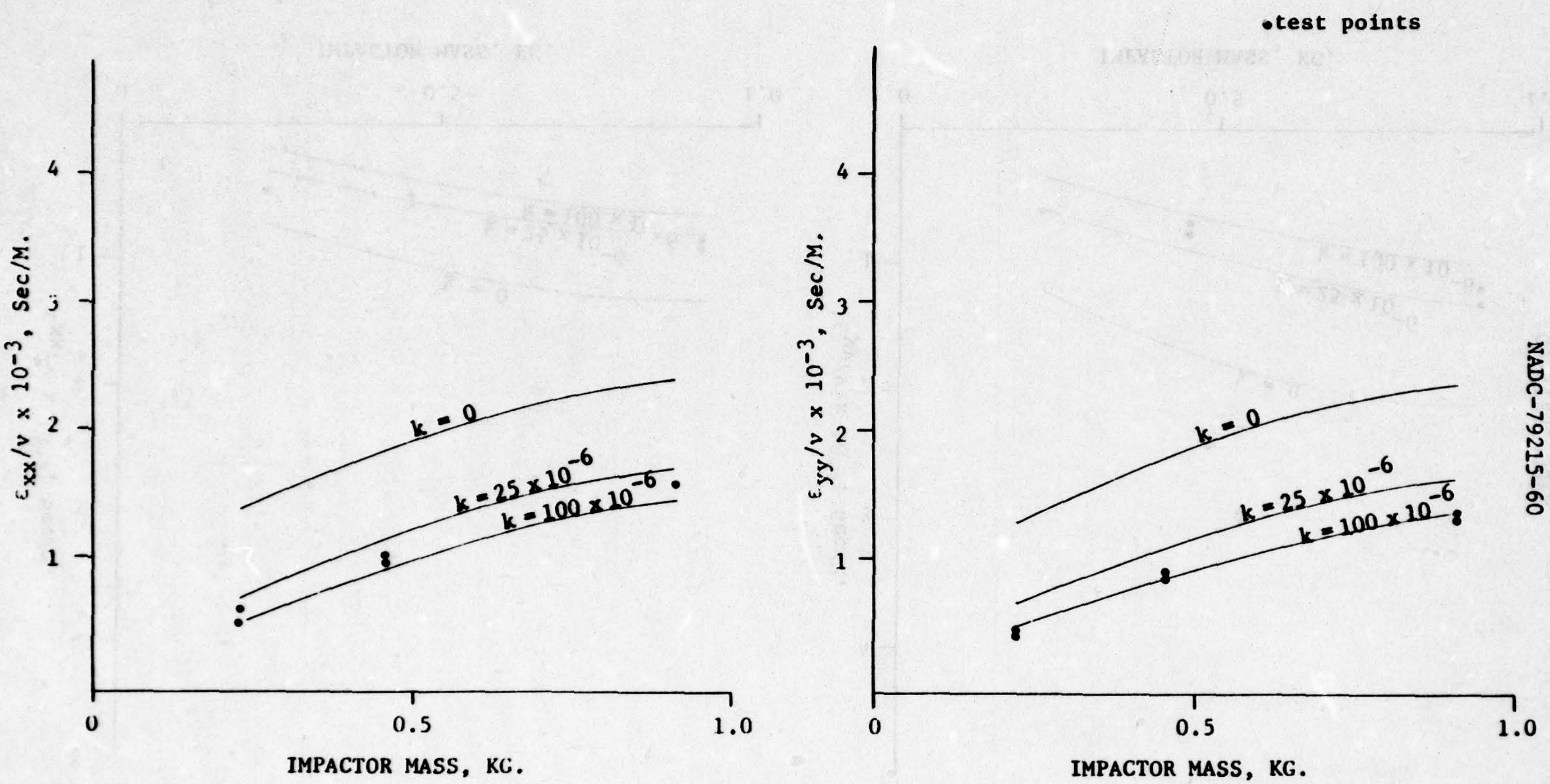


Figure 14 - Comparison Between Theoretical and Experimental Results for Plate Series H1

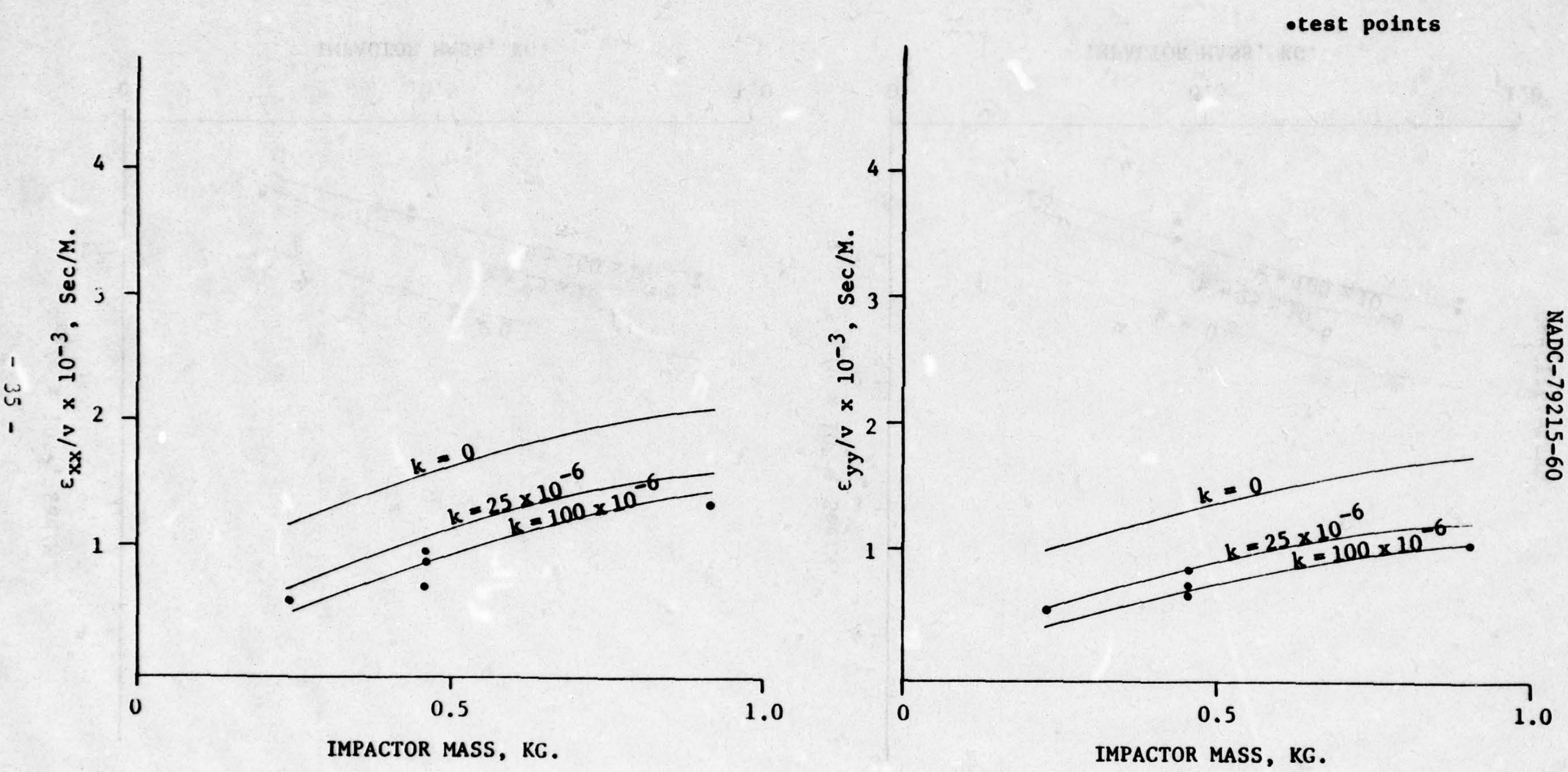


Figure 15 - Comparison Between Theoretical and Experimental Results for Plate Series H2

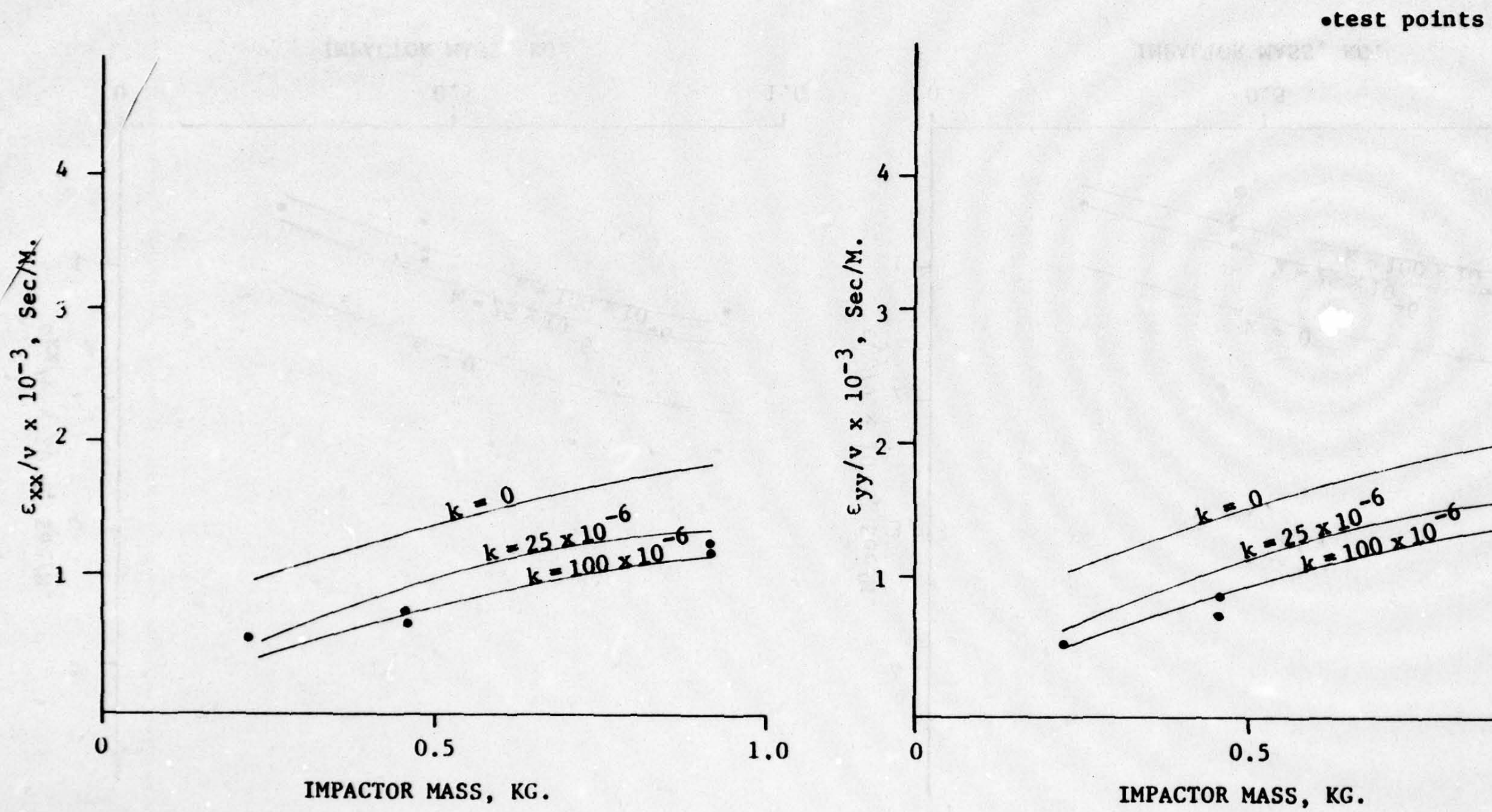


Figure 16 - Comparison Between Theoretical and Experimental Results for Plate Series H3

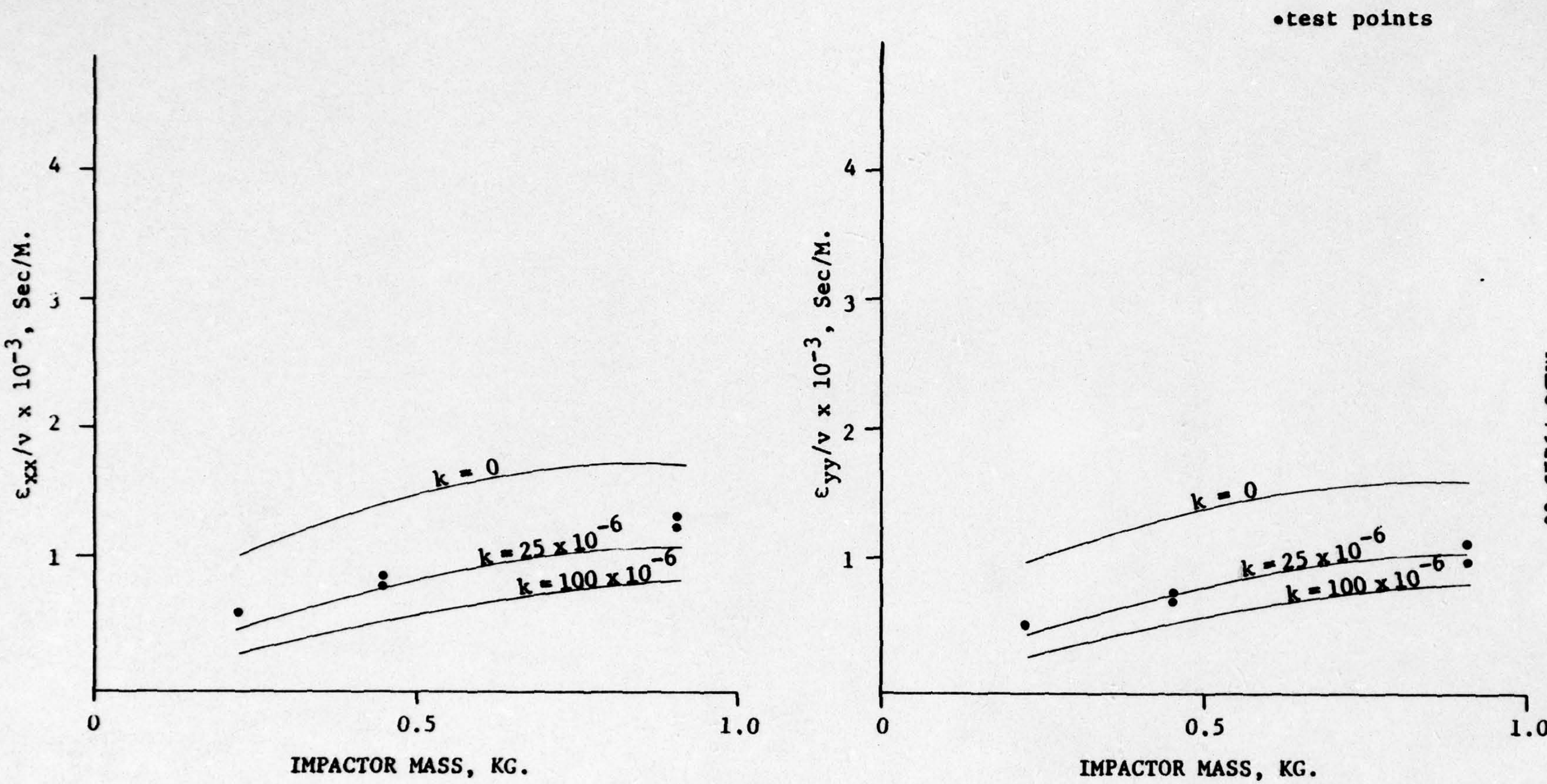


Figure 17 - Comparison Between Theoretical and Experimental Results for Plate Series H4

APPENDIX A

FUNCTIONS NEEDED FOR THE EVALUATION OF THE SOLUTION

GENERAL REPRESENTATION OF TIME TRANSFORMED FUNCTIONS

Note that functions defined in the text are not repeated in this appendix.

$$\frac{db_r}{dp} = \frac{1}{4b_r} \cdot \frac{\rho h}{D_x}\left[\frac{\rho h}{D_x} p^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{-1/2} p$$

$$\frac{db_i}{dp} = \frac{1}{4b_i} \cdot \frac{\rho h}{D_x}\left[\frac{\rho h}{D_x} p^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{-1/2} p$$

$$\begin{aligned}\frac{d\Omega_s}{dp} = (a-c+\epsilon_1)\Big[&\cosh b_r(a-c+\epsilon_1)\sin b_i(a-c+\epsilon_1)\frac{db_r}{dp} \\ &+ \sinh b_r(a-c+\epsilon_1)\cos b_i(a-c+\epsilon_1)\frac{db_i}{dp}\Big] \\ -(a-c-\epsilon_1)\Big[&\cosh b_r(a-c-\epsilon_1)\sin b_i(a-c-\epsilon_1)\frac{db_r}{dp} \\ &+ \sinh b_r(a-c-\epsilon_1)\cos b_i(a-c-\epsilon_1)\frac{db_i}{dp}\Big]\end{aligned}$$

$$\begin{aligned}\frac{d\Omega_c}{dp} = (a-c+\epsilon_1)\Big[&\sinh b_r(a-c+\epsilon_1)\cos b_i(a-c+\epsilon_1)\frac{db_r}{dp} \\ &- \cosh b_r(a-c+\epsilon_1)\sin b_i(a-c+\epsilon_1)\frac{db_i}{dp}\Big] \\ -(a-c-\epsilon_1)\Big[&\sinh b_r(a-c-\epsilon_1)\cos b_i(a-c-\epsilon_1)\frac{db_r}{dp} \\ &- \cosh b_r(a-c-\epsilon_1)\sin b_i(a-c-\epsilon_1)\frac{db_i}{dp}\Big]\end{aligned}$$

$$\begin{aligned}\frac{d\Omega_o}{dp} = &\left[\frac{db_r}{dp}\Omega_s - \frac{db_i}{dp}\Omega_c + b_r\frac{d\Omega_s}{dp} - b_i\frac{d\Omega_c}{dp}\right]\sinh b_r a\cos b_i a \\ &+ a(b_r\Omega_s - b_i\Omega_c)\left(\cosh b_r a\cos b_i a\frac{db_r}{dp} - \sinh b_r a\sin b_i a\frac{db_i}{dp}\right) \\ &- \left[\frac{db_i}{dp}\Omega_s + \frac{db_r}{dp}\Omega_c + b_i\frac{d\Omega_s}{dp} + b_r\frac{d\Omega_c}{dp}\right]\cosh b_r a\sin b_i a \\ &- a(b_i\Omega_s + b_r\Omega_c)\left(\sinh b_r a\sin b_i a\frac{db_r}{dp} + \cosh b_r a\cos b_i a\frac{db_i}{dp}\right)\end{aligned}$$

$$\frac{d\Omega_1}{dp} = \left[\frac{db_r}{dp}\Omega_s + \frac{db_i}{dp}\Omega_c + b_r\frac{d\Omega_s}{dp} + b_i\frac{d\Omega_c}{dp}\right]\sinh b_r a \cos b_i a$$

$$+ a(b_r\Omega_s + b_i\Omega_c)\left(\cosh b_r a \cos b_i a \frac{db_r}{dp} - \sinh b_r a \sin b_i a \frac{db_i}{dp}\right)$$

$$+ \left[\frac{db_i}{dp}\Omega_s - \frac{db_r}{dp}\Omega_c + b_i\frac{d\Omega_s}{dp} - b_r\frac{d\Omega_c}{dp}\right]\cosh b_r a \sin b_i a$$

$$+ a(b_i\Omega_s - b_r\Omega_c)\left(\sinh b_r a \sin b_i a \frac{db_r}{dp} + \cosh b_r a \cos b_i a \frac{db_i}{dp}\right)$$

$$\frac{d\lambda_m}{dp} = -\frac{2}{m b_r^3 b_i^3 (b_r^2+b_i^2)^3}\sin\frac{\pi m \epsilon_2}{b}\sin^2\frac{\pi m d}{b}\,\cdot$$

$$\cdot\left[b_i(3b_r^2+b_i^2)\frac{db_r}{dp} + b_r(b_r^2+3b_i^2)\frac{db_i}{dp}\right]$$

$$\frac{d\nu_m}{dp} = 2\left\{\left[b_i(3b_r^2-b_i^2)\frac{db_r}{dp} + b_r(b_r^2-3b_i^2)\frac{db_i}{dp}\right]\sinh b_r\epsilon_1 \sin b_i\epsilon_1\right.$$

$$+ 4b_r b_i\left(b_i\frac{db_r}{dp} + b_r\frac{db_i}{dp}\right)(1-\cosh b_r\epsilon_1\cos b_i\epsilon_1)$$

$$+ \epsilon_1 b_r b_i(b_r^2-b_i^2)\left[\cosh b_r\epsilon_1 \sin b_i\epsilon_1\frac{db_r}{dp} + \sinh b_r\epsilon_1\cos b_i\epsilon_1\frac{db_i}{dp}\right]$$

$$\left.+ 2\epsilon_1 b_r^2 b_i^2\left[-\sinh b_r\epsilon_1\cos b_i\epsilon_1\frac{db_r}{dp} + \cosh b_r\epsilon_1\sin b_i\epsilon_1\frac{db_i}{dp}\right]\right\}$$

$$\frac{d\mu_m}{dp} = \frac{a}{(\sinh^2 b_r a + \sin^2 b_i a)^2} \cdot \left(\sinh 2 b_r a \frac{db_r}{dp} + \sin 2 b_i a \frac{db_i}{dp} \right) \cdot$$

$$\cdot \left\{ b_r \left[(-b_r^2 + 3 b_i^2) \Omega_0 + (b_r^2 + b_i^2) \Omega_1 \right] \cosh b_r c \sin b_i c \right.$$

$$\left. + b_i \left[(3 b_r^2 - b_i^2) \Omega_0 - (b_r^2 + b_i^2) \Omega_1 \right] \sinh b_r c \cos b_i c \right\}$$

$$- \frac{1}{(\sinh^2 b_r a + \sin^2 b_i a)} \cdot$$

$$\cdot \left\{ \left[\left(-3 b_r^2 \frac{db_r}{dp} + 3 b_i^2 \frac{db_r}{dp} + 6 b_r b_i \frac{db_i}{dp} \right) \Omega_0 \right. \right.$$

$$\left. + \left(3 b_r^2 \frac{db_r}{dp} + b_i^2 \frac{db_r}{dp} + 2 b_r b_i \frac{db_i}{dp} \right) \Omega_1 \right] \cosh b_r c \sin b_i c$$

$$+ b_r \left[(-b_r^2 + 3 b_i^2) \frac{d\Omega_0}{dp} + (b_r^2 + b_i^2) \frac{d\Omega_1}{dp} \right] \cosh b_r c \sin b_i c$$

$$+ \left[\left(6 b_r b_i \frac{db_r}{dp} + 3 b_r^2 \frac{db_i}{dp} - 3 b_i^2 \frac{db_i}{dp} \right) \Omega_0 \right.$$

$$\left. + \left(-2 b_r b_i \frac{db_r}{dp} - b_r^2 \frac{db_i}{dp} - 3 b_i^2 \frac{db_i}{dp} \right) \Omega_1 \right] \sinh b_r c \cos b_i c$$

$$+ b_i \left[(3 b_r^2 - b_i^2) \frac{d\Omega_0}{dp} - (b_r^2 + b_i^2) \frac{d\Omega_1}{dp} \right] \sinh b_r c \cos b_i c$$

$$+ c\, b_r \left[(-b_r^2 + 3 b_i^2) \Omega_0 + (b_r^2 + b_i^2) \Omega_1 \right] \cdot$$

$$\cdot \left[\sinh b_r c \sin b_i c \frac{db_r}{dp} + \cosh b_r c \cos b_i c \frac{db_i}{dp} \right]$$

$$+ c\, b_i \left[(3 b_r^2 - b_i^2) \Omega_0 - (b_r^2 + b_i^2) \Omega_1 \right] \cdot$$

$$\left. \cdot \left[\cosh b_r c \cos b_i c \frac{db_r}{dp} - \sinh b_r c \sin b_i c \frac{db_i}{dp} \right] \right\}$$

REPRESENTATION OF TIME TRANSFORMED FUNCTIONS CORRESPONDING TO SPECIFIC SUBCASES:

Case 1 -

$$b_r = i b_r^*$$

$$b_i = i b_i^*$$

$$\Omega_s = -\left[\sin b_r^*(a-c+\epsilon_1)\sinh b_i^*(a-c+\epsilon_1) - \sin b_r^*(a-c-\epsilon_1)\sinh b_i^*(a-c-\epsilon_1)\right]$$

$$\Omega_c = \cos b_r^*(a-c+\epsilon_1)\cosh b_i^*(a-c+\epsilon_1) - \cos b_r^*(a-c-\epsilon_1)\cosh b_i^*(a-c-\epsilon_1)$$

$$\Omega_0 = -\left[(b_r^*\Omega_s - b_i^*\Omega_c)\sin b_r^* a\cosh b_i^* a - (b_i^*\Omega_s + b_r^*\Omega_c)\cos b_r^* a\sinh b_i^* a\right]$$

$$\Omega_1 = -\left[(b_r^*\Omega_s + b_i^*\Omega_c)\sin b_r^* a\cosh b_i^* a + (b_i^*\Omega_s - b_r^*\Omega_c)\cos b_r^* a\sinh b_i^* a\right]$$

$$\xi_m = \frac{1}{m b_r^{*2} b_i^{*2}(b_r^{*2}+b_i^{*2})^2}\sin\frac{\pi m\epsilon_3}{b}\sin\frac{\pi m d}{b}\sin\frac{\pi m y}{b}\cdot$$

$$\cdot\left\{\frac{1}{\sin^2 b_r^* a + \sinh^2 b_i^* a}\cdot\right.$$

$$\cdot\Big\langle b_r^*\left[(-b_r^{*2}+3b_i^{*2})\Omega_0 + (b_r^{*2}+b_i^{*2})\Omega_1\right]\cos b_r^* x\sinh b_i^* x$$

$$+ b_i^*\left[(3b_r^{*2}-b_i^{*2})\Omega_0 - (b_r^{*2}+b_i^{*2})\Omega_1\right]\sin b_r^* x\cosh b_i^* x\Big\rangle$$

$$- 2b_r^* b_i^*\Big\langle\Big[(b_r^{*2}-b_i^{*2})\sin b_r^*(x-c+\epsilon_1)\sinh b_i^*(x-c+\epsilon_1)$$

$$-2b_r^* b_i^*\left\{1-\cos b_r^*(x-c+\epsilon_1)\cosh b_i^*(x-c+\epsilon_1)\right\}\Big]U(x-c+\epsilon_1)$$

$$-\Big[(b_r^{*2}-b_i^{*2})\sin b_r^*(x-c-\epsilon_1)\sinh b_i^*(x-c-\epsilon_1)$$

$$\left.-2b_r^* b_i^*\left\{1-\cos b_r^*(x-c-\epsilon_1)\cosh b_i^*(x-c-\epsilon_1)\right\}\Big]U(x-c-\epsilon_1)\Big\rangle\right\}$$

$$\lambda_m = \frac{1}{m b_r^{*2} b_i^{*2}(b_r^{*2}+b_i^{*2})^2}\sin\frac{\pi m\epsilon_2}{b}\sin^2\frac{\pi m d}{b}$$

$$\mu_m = \frac{1}{\sin^2 b_r^* a + \sinh^2 b_i^* a} \cdot \Big\{ b_r^* \big[(-b_r^{*2} + 3 b_i^{*2}) \Omega_0 + (b_r^{*2} + b_i^{*2}) \Omega_1 \big] \cos b_r^* c \sinh b_i^* c$$

$$+ b_i^* \big[(3 b_r^{*2} - b_i^{*2}) \Omega_0 - (b_r^{*2} + b_i^{*2}) \Omega_1 \big] \sin b_r^* c \cosh b_i^* c \Big\}$$

$$\nu_m = -2 b_r^* b_i^* \Big\{ (b_r^{*2} - b_i^{*2}) \sin b_r^* \epsilon_1 \sinh b_i^* \epsilon_1$$

$$- 2 b_r^* b_i^* (1 - \cos b_r^* \epsilon_1 \cosh b_i^* \epsilon_1) \Big\}$$

$$\frac{db_r}{dp} = -\frac{1}{4 b_r^*} \cdot \frac{\rho h}{D_x} \left[-\frac{\rho h}{D_x} p_y^2 + \frac{D_y}{D_x} \left(\frac{\pi m}{b}\right)^4 \right]^{-1/2} p_y$$

$$\frac{db_i}{dp} = -\frac{1}{4 b_i^*} \cdot \frac{\rho h}{D_x} \left[-\frac{\rho h}{D_x} p_y^2 + \frac{D_y}{D_x} \left(\frac{\pi m}{b}\right)^4 \right]^{-1/2} p_y$$

$$\frac{d\Omega_s}{dp} = i \left(\frac{d\Omega_s}{dp}\right)^*$$

$$\left(\frac{d\Omega_s}{dp}\right)^* = (a - c + \epsilon_1) \Big[\cos b_r^* (a - c + \epsilon_1) \sinh b_i^* (a - c + \epsilon_1) \frac{db_r}{dp}$$

$$+ \sin b_r^* (a - c + \epsilon_1) \cosh b_i^* (a - c + \epsilon_1) \frac{db_i}{dp} \Big]$$

$$- (a - c - \epsilon_1) \Big[\cos b_r^* (a - c - \epsilon_1) \sinh b_i^* (a - c - \epsilon_1) \frac{db_r}{dp}$$

$$+ \sin b_r^* (a - c - \epsilon_1) \cosh b_i^* (a - c - \epsilon_1) \frac{db_i}{dp} \Big]$$

$$\frac{d\Omega_c}{dp} = i \left(\frac{d\Omega_c}{dp}\right)^*$$

$$\left(\frac{d\Omega_c}{dp}\right)^* = (a - c + \epsilon_1) \Big[\sin b_r^* (a - c + \epsilon_1) \cosh b_i^* (a - c + \epsilon_1) \frac{db_r}{dp}$$

$$- \cos b_r^* (a - c + \epsilon_1) \sinh b_i^* (a - c + \epsilon_1) \frac{db_i}{dp} \Big]$$

$$- (a - c - \epsilon_1) \Big[\sin b_r^* (a - c - \epsilon_1) \cosh b_i^* (a - c - \epsilon_1) \frac{db_r}{dp}$$

$$- \cos b_r^* (a - c - \epsilon_1) \sinh b_i^* (a - c - \epsilon_1) \frac{db_i}{dp} \Big]$$

$$\frac{d\mathcal{R}_0}{dp} = i\left(\frac{d\mathcal{R}_0}{dp}\right)^*$$

$$\left(\frac{d\mathcal{R}_0}{dp}\right)^* = \left[\frac{db_r}{dp}\mathcal{R}_s - \frac{db_i}{dp}\mathcal{R}_c - b_r^+\left(\frac{d\mathcal{R}_s}{dp}\right)^* + b_i^+\left(\frac{d\mathcal{R}_c}{dp}\right)^*\right]\sin b_r^+a\cosh b_i^+a$$

$$+ a(b_r^+\mathcal{R}_s - b_i^+\mathcal{R}_c)\left(\cos b_r^+a\cosh b_i^+a\frac{db_r}{dp} + \sin b_r^+a\sinh b_i^+a\frac{db_i}{dp}\right)$$

$$-\left[\frac{db_i}{dp}\mathcal{R}_s + \frac{db_r}{dp}\mathcal{R}_c - b_i^+\left(\frac{d\mathcal{R}_s}{dp}\right)^* - b_r^+\left(\frac{d\mathcal{R}_c}{dp}\right)^*\right]\cos b_r^+a\sinh b_i^+a$$

$$- a(b_i^+\mathcal{R}_s + b_r^+\mathcal{R}_c)\left(-\sin b_r^+a\sinh b_i^+a\frac{db_r}{dp} + \cos b_r^+a\cosh b_i^+a\frac{db_i}{dp}\right)$$

$$\frac{d\mathcal{R}_1}{dp} = i\left(\frac{d\mathcal{R}_1}{dp}\right)^*$$

$$\left(\frac{d\mathcal{R}_1}{dp}\right)^* = \left[\frac{db_r}{dp}\mathcal{R}_s + \frac{db_i}{dp}\mathcal{R}_c - b_r^+\left(\frac{d\mathcal{R}_s}{dp}\right)^* - b_i^+\left(\frac{d\mathcal{R}_c}{dp}\right)^*\right]\sin b_r^+a\cosh b_i^+a$$

$$+ a(b_r^+\mathcal{R}_s + b_i^+\mathcal{R}_c)\left(\cos b_r^+a\cosh b_i^+a\frac{db_r}{dp} + \sin b_r^+a\sinh b_i^+a\frac{db_i}{dp}\right)$$

$$+\left[\frac{db_i}{dp}\mathcal{R}_s - \frac{db_r}{dp}\mathcal{R}_c - b_i^+\left(\frac{d\mathcal{R}_s}{dp}\right)^* + b_r^+\left(\frac{d\mathcal{R}_c}{dp}\right)^*\right]\cos b_r^+a\sinh b_i^+a$$

$$+ a(b_i^+\mathcal{R}_s - b_r^+\mathcal{R}_c)\left(-\sin b_r^+a\sinh b_i^+a\frac{db_r}{dp} + \cos b_r^+a\cosh b_i^+a\frac{db_i}{dp}\right)$$

$$\frac{d\lambda_m}{dp} = i\left(\frac{d\lambda_m}{dp}\right)^*$$

$$\left(\frac{d\lambda_m}{dp}\right)^* = \frac{2}{m\left[b_r^+b_i^+(b_r^{+2}+b_i^{+2})\right]^3}\sin\frac{\pi m\epsilon_2}{b}\sin^2\frac{\pi m d}{b}\,\cdot$$

$$\cdot\left\{b_i^+(3b_r^{+2}+b_i^{+2})\frac{db_r}{dp} + b_r^+(b_r^{+2}+3b_i^{+2})\frac{db_i}{dp}\right\}$$

$$\frac{d\nu_m}{dp} = i\left(\frac{d\nu_m}{dp}\right)^*$$

$$\left(\frac{d\nu_m}{dp}\right)^* = 2\left\{\left[b_i^+(3b_r^{+2}-b_i^{+2})\frac{db_r}{dp} + b_r^+(b_r^{+2}-3b_i^{+2})\frac{db_i}{dp}\right]\sin b_r^+\epsilon_1\sinh b_i^+\epsilon_1\right.$$

$$-4b_r^+b_i^+\left(b_i^+\frac{db_r}{dp} + b_r^+\frac{db_i}{dp}\right)\left(1-\cos b_r^+\epsilon_1\cosh b_i^+\epsilon_1\right)$$

$$+\epsilon_1 b_r^+b_i^+(b_r^{+2}-b_i^{+2})\left[\cos b_r^+\epsilon_1\sinh b_i^+\epsilon_1\frac{db_r}{dp} + \sin b_r^+\epsilon_1\cosh b_i^+\epsilon_1\frac{db_i}{dp}\right]$$

$$\left.+2\epsilon_1 b_r^{+2}b_i^{+2}\left[-\sin b_r^+\epsilon_1\cosh b_i^+\epsilon_1\frac{db_r}{dp} + \cos b_r^+\epsilon_1\sinh b_i^+\epsilon_1\frac{db_i}{dp}\right]\right\}$$

$$\frac{d\mu_m}{dp} = i\left(\frac{d\mu_m}{dp}\right)^*$$

$$\left(\frac{d\mu_m}{dp}\right)^* = \frac{a}{[\sin^2 b_r^+ a + \sinh^2 b_i^+ a]^2} \cdot \left(\sin 2b_r^+ a \frac{db_r}{dp} + \sinh 2b_i^+ a \frac{db_i}{dp}\right) \cdot$$

$$\cdot \Big\{ b_r^+ \left[(-b_r^{+2} + 3b_i^{+2})\Omega_0 + (b_r^{+2} + b_i^{+2})\Omega_1 \right] \cos b_r^+ c \sinh b_i^+ c$$

$$+ b_i^+ \left[(3b_r^{+2} - b_i^{+2})\Omega_0 - (b_r^{+2} + b_i^{+2})\Omega_1 \right] \sin b_r^+ c \cosh b_i^+ c \Big\}$$

$$+ \frac{1}{\sin^2 b_r^+ a + \sinh^2 b_i^+ a} \cdot$$

$$\cdot \Big\{ -\Big[3\left(-b_r^{+2}\frac{db_r}{dp} + b_i^{+2}\frac{db_r}{dp} + 2b_r^+ b_i^+ \frac{db_i}{dp}\right)\Omega_0$$

$$+ \left(3b_r^{+2}\frac{db_r}{dp} + b_i^{+2}\frac{db_r}{dp} + 2b_r^+ b_i^+ \frac{db_i}{dp}\right)\Omega_1 \Big] \cos b_r^+ c \sinh b_i^+ c$$

$$+ b_r^+ \left[(-b_r^{+2} + 3b_i^{+2})\left(\frac{d\Omega_0}{dp}\right)^* + (b_r^{+2} + b_i^{+2})\left(\frac{d\Omega_1}{dp}\right)^* \right] \cos b_r^+ c \sinh b_i^+ c$$

$$- \Big[3\left(2b_r^+ b_i^+ \frac{db_r}{dp} + b_r^{+2}\frac{db_i}{dp} - b_i^{+2}\frac{db_i}{dp}\right)\Omega_0$$

$$+ \left(-2b_r^+ b_i^+ \frac{db_r}{dp} - b_r^{+2}\frac{db_i}{dp} - 3b_i^{+2}\frac{db_i}{dp}\right)\Omega_1 \Big] \sin b_r^+ c \cosh b_i^+ c$$

$$+ b_i^+ \left[(3b_r^{+2} - b_i^{+2})\left(\frac{d\Omega_0}{dp}\right)^* - (b_r^{+2} + b_i^{+2})\left(\frac{d\Omega_1}{dp}\right)^* \right] \sin b_r^+ c \cosh b_i^+ c$$

$$- c b_r^+ \left[(-b_r^{+2} + 3b_i^{+2})\Omega_0 + (b_r^{+2} + b_i^{+2})\Omega_1 \right] \cdot$$

$$\cdot \left[-\sin b_r^+ c \sinh b_i^+ c \frac{db_r}{dp} + \cos b_r^+ c \cosh b_i^+ c \frac{db_i}{dp} \right]$$

$$- c b_i^+ \left[(3b_r^{+2} - b_i^{+2})\Omega_0 - (b_r^{+2} + b_i^{+2})\Omega_1 \right] \cdot$$

$$\cdot \left[\cos b_r^+ c \cosh b_i^+ c \frac{db_r}{dp} + \sin b_r^+ c \sinh b_i^+ c \frac{db_i}{dp} \right] \Big\}$$

Case 2 -

$$b_i = i\, b_i^*$$

$$\Omega_s = i\,\Omega_s^*$$

$$\Omega_s^* = \sinh b_r(a-c+\epsilon_1)\sinh b_i^*(a-c+\epsilon_1) - \sinh b_r(a-c-\epsilon_1)\sinh b_i^*(a-c-\epsilon_1)$$

$$\Omega_c = \cosh b_r(a-c+\epsilon_1)\cosh b_i^*(a-c+\epsilon_1) - \cosh b_r(a-c-\epsilon_1)\cosh b_i^*(a-c-\epsilon_1)$$

$$\Omega_o = i\,\Omega_o^*$$

$$\Omega_o^* = (b_r\Omega_s^* - b_i^*\Omega_c)\sinh b_r a \cosh b_i^* a - (-b_i^*\Omega_s^* + b_r\Omega_c)\cosh b_r a \sinh b_i^* a$$

$$\Omega_1 = i\,\Omega_1^*$$

$$\Omega_1^* = (b_r\Omega_s^* + b_i^*\Omega_c)\sinh b_r a \cosh b_i^* a - (b_i^*\Omega_s^* + b_r\Omega_c)\cosh b_r a \sinh b_i^* a$$

$$\begin{aligned}
\xi_m = -\frac{1}{m b_r^2 b_i^{*2}(b_r^2 - b_i^{*2})^2} \sin\frac{\pi m \epsilon_2}{b}\sin\frac{\pi m d}{b}\sin\frac{\pi m y}{b}\,\cdot \\
\cdot\Bigg\{ -\frac{1}{\sinh^2 b_r a - \sinh^2 b_i^* a}\,\cdot \\
\cdot\Big\langle -b_r\left[-(b_r^2+3b_i^{*2})\Omega_o^* + (b_r^2-b_i^{*2})\Omega_1^*\right]\cosh b_r x \sinh b_i^* x \\
- b_i^*\left[(3b_r^2+b_i^{*2})\Omega_o^* - (b_r^2-b_i^{*2})\Omega_1^*\right]\sinh b_r x \cosh b_i^* x\Big\rangle \\
-2b_r b_i^*\Big\langle\Big[(b_r^2+b_i^{*2})\sinh b_r(x-c+\epsilon_1)\sinh b_i^*(x-c+\epsilon_1) \\
+2b_r b_i^*\left\{1-\cosh b_r(x-c+\epsilon_1)\cosh b_i^*(x-c+\epsilon_1)\right\}\Big]U(x-c+\epsilon_1) \\
-\Big[(b_r^2+b_i^{*2})\sinh b_r(x-c-\epsilon_1)\sinh b_i^*(x-c-\epsilon_1) \\
+2b_r b_i^*\left\{1-\cosh b_r(x-c-\epsilon_1)\cosh b_i^*(x-c-\epsilon_1)\right\}\Big]U(x-c-\epsilon_1)\Big\rangle\Bigg\}
\end{aligned}$$

$$\lambda_m = -\frac{1}{m k_r^2 k_i^{*2} (k_r^2 - k_i^{*2})^2} \sin\frac{\pi m \epsilon_2}{b} \sin^2\frac{\pi m d}{b}$$

$$\mu_m = \frac{1}{\sinh^2 k_r a - \sinh^2 k_i^* a} \cdot$$

$$\cdot \Big\{ k_r\left[-(k_r^2 + 3k_i^{*2})\Omega_0^* + (k_r^2 - k_i^{*2})\Omega_1^*\right] \cosh k_r c \sinh k_i^* c$$

$$+ k_i^*\left[(3k_r^2 + k_i^{*2})\Omega_0^* - (k_r^2 - k_i^{*2})\Omega_1^*\right] \sinh k_r c \cosh k_i^* c \Big\}$$

$$\nu_m = -2k_r k_i^* \Big\{ (k_r^2 + k_i^{*2}) \sinh k_r \epsilon_1 \sinh k_i^* \epsilon_1 + 2k_r k_i^* (1 - \cosh k_r \epsilon_1 \cosh k_i^* \epsilon_1) \Big\}$$

$$\frac{dk_r}{dp} = i\left(\frac{dk_r}{dp}\right)^*$$

$$\left(\frac{dk_r}{dp}\right)^* = -\frac{1}{4k_r} \cdot \frac{\rho h}{D_x}\left[-\frac{\rho h}{D_x} p_y^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{-1/2} p_y$$

$$\frac{dk_i}{dp} = -\frac{1}{4k_i^*} \cdot \frac{\rho h}{D_x}\left[-\frac{\rho h}{D_x} p_y^2 + \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{-1/2} p_y$$

$$\frac{d\Omega_s}{dp} = (a-c+\epsilon_1)\Big[-\cosh k_r(a-c+\epsilon_1) \sinh k_i^*(a-c+\epsilon_1)\left(\frac{dk_r}{dp}\right)^*$$

$$+ \sinh k_r(a-c+\epsilon_1) \cosh k_i^*(a-c+\epsilon_1) \frac{dk_i}{dp}\Big]$$

$$-(a-c-\epsilon_1)\Big[-\cosh k_r(a-c-\epsilon_1) \sinh k_i^*(a-c-\epsilon_1)\left(\frac{dk_r}{dp}\right)^*$$

$$+ \sinh k_r(a-c-\epsilon_1) \cosh k_i^*(a-c-\epsilon_1) \frac{dk_i}{dp}\Big]$$

$$\frac{d\Omega_c}{dp} = i\left(\frac{d\Omega_c}{dp}\right)^*$$

$$\left(\frac{d\Omega_c}{dp}\right)^* = (a-c+\epsilon_1)\Big[\sinh k_r(a-c+\epsilon_1) \cosh k_i^*(a-c+\epsilon_1)\left(\frac{dk_r}{dp}\right)^*$$

$$- \cosh k_r(a-c+\epsilon_1) \sinh k_i^*(a-c+\epsilon_1) \frac{dk_i}{dp}\Big]$$

$$-(a-c-\epsilon_1)\Big[\sinh k_r(a-c-\epsilon_1) \cosh k_i^*(a-c-\epsilon_1)\left(\frac{dk_r}{dp}\right)^*$$

$$- \cosh k_r(a-c-\epsilon_1) \sinh k_i^*(a-c-\epsilon_1) \frac{dk_i}{dp}\Big]$$

$$\frac{d\Omega_0}{dp} = \left[-\left(\frac{db_r}{dp}\right)^* \Omega_s^* - \frac{db_i}{dp}\Omega_c + b_r\frac{d\Omega_s}{dp} + b_i^*\left(\frac{d\Omega_c}{dp}\right)^*\right] \sinh b_r a \cosh b_i^* a$$

$$- a\left(b_r\Omega_s^* - b_i^*\Omega_c\right)\left[\cosh b_r a \cosh b_i^* a \left(\frac{db_r}{dp}\right)^* - \sinh b_r a \sinh b_i^* a \frac{db_i}{dp}\right]$$

$$+ \left[\frac{db_i}{dp}\Omega_s^* + \left(\frac{db_r}{dp}\right)^*\Omega_c + b_i^*\frac{d\Omega_s}{dp} + b_r\left(\frac{d\Omega_c}{dp}\right)^*\right] \cosh b_r a \sinh b_i^* a$$

$$- a\left(-b_i^*\Omega_s^* + b_r\Omega_c\right)\left[-\sinh b_r a \sinh b_i^* a \left(\frac{db_r}{dp}\right)^* + \cosh b_r a \cosh b_i^* a \frac{db_i}{dp}\right]$$

$$\frac{d\Omega_1}{dp} = \left[-\left(\frac{db_r}{dp}\right)^* \Omega_s^* + \frac{db_i}{dp}\Omega_c + b_r\frac{d\Omega_s}{dp} - b_i^*\left(\frac{d\Omega_c}{dp}\right)^*\right] \sinh b_r a \cosh b_i^* a$$

$$- a\left(b_r\Omega_s^* + b_i^*\Omega_c\right)\left[\cosh b_r a \cosh b_i^* a \left(\frac{db_r}{dp}\right)^* - \sinh b_r a \sinh b_i^* a \frac{db_i}{dp}\right]$$

$$- \left[\frac{db_i}{dp}\Omega_s^* - \left(\frac{db_r}{dp}\right)^*\Omega_c + b_i^*\frac{d\Omega_s}{dp} - b_r\left(\frac{d\Omega_c}{dp}\right)^*\right] \cosh b_r a \sinh b_i^* a$$

$$- a\left(b_i^*\Omega_s^* + b_r\Omega_c\right)\left[-\sinh b_r a \sinh b_i^* a \left(\frac{db_r}{dp}\right)^* + \cosh b_r a \cosh b_i^* a \frac{db_i}{dp}\right]$$

$$\frac{d\lambda_m}{dp} = i\left(\frac{d\lambda_m}{dp}\right)^*$$

$$\left(\frac{d\lambda_m}{dp}\right)^* = -\frac{2}{m\left[b_r b_i^*\left(b_r^2 - b_i^{*2}\right)\right]^3} \sin\frac{\pi m \epsilon_2}{b} \sin^2\frac{\pi m d}{b} \cdot$$

$$\cdot \left\{-b_i^*\left(3b_r^2 - b_i^{*2}\right)\left(\frac{db_r}{dp}\right)^* + b_r\left(b_r^2 - 3b_i^{*2}\right)\frac{db_i}{dp}\right\}$$

$$\frac{d\nu_m}{dp} = i\left(\frac{d\nu_m}{dp}\right)^*$$

$$\left(\frac{d\nu_m}{dp}\right)^* = 2\left\{\left[-b_i^*\left(3b_r^2 + b_i^{*2}\right)\left(\frac{db_r}{dp}\right)^* + b_r\left(b_r^2 + 3b_i^{*2}\right)\frac{db_i}{dp}\right] \sinh b_r\epsilon_1 \sinh b_i^*\epsilon_1\right.$$

$$+ 4b_r b_i^*\left[-b_i^*\left(\frac{db_r}{dp}\right)^* + b_r\frac{db_i}{dp}\right]\left(1 - \cosh b_r\epsilon_1 \cosh b_i^*\epsilon_1\right)$$

$$+ \epsilon_1 b_r b_i^*\left(b_r^2 + b_i^{*2}\right)\left[-\cosh b_r\epsilon_1 \sinh b_i^*\epsilon_1 \left(\frac{db_r}{dp}\right)^* + \sinh b_r\epsilon_1 \cosh b_i^*\epsilon_1 \frac{db_i}{dp}\right]$$

$$\left.- 2\epsilon_1 b_r^2 b_i^{*2}\left[-\sinh b_r\epsilon_1 \cosh b_i^*\epsilon_1 \left(\frac{db_r}{dp}\right)^* + \cosh b_r\epsilon_1 \sinh b_i^*\epsilon_1 \frac{db_i}{dp}\right]\right\}$$

$$\frac{d\mu_m}{dp} = i\left(\frac{d\mu_m}{dp}\right)^*$$

$$\left(\frac{d\mu_m}{dp}\right)^* = \frac{a}{[\sinh^2 b_r a - \sinh^2 b_i^* a]^2} \cdot \left[\sinh 2b_r a \left(\frac{db_r}{dp}\right)^* + \sinh 2b_i^* a \frac{db_i}{dp}\right] \cdot$$

$$\cdot \Big\{ b_r\left[(b_r^2 + 3b_i^{*2})\Omega_0^* - (b_r^2 - b_i^{*2})\Omega_1^*\right] \cosh b_r c \sinh b_i^* c$$

$$+ b_i^*\left[-(3b_r^2 + b_i^{*2})\Omega_0^* + (b_r^2 - b_i^{*2})\Omega_1^*\right] \sinh b_r c \cosh b_i^* c \Big\}$$

$$- \frac{1}{\sinh^2 b_r a - \sinh^2 b_i^* a} \cdot$$

$$\cdot \Big\{ \Big[3\left\langle b_r^2 \left(\frac{db_r}{dp}\right)^* + b_i^{*2}\left(\frac{db_r}{dp}\right)^* - 2b_r b_i^* \frac{db_i}{dp}\right\rangle \Omega_0^*$$

$$- \left\langle 3b_r^2\left(\frac{db_r}{dp}\right)^* - b_i^{*2}\left(\frac{db_r}{dp}\right)^* + 2b_r b_i^* \frac{db_i}{dp}\right\rangle \Omega_1^* \Big] \cosh b_r c \sinh b_i^* c$$

$$+ b_r\left[-(b_r^2 + 3b_i^{*2})\frac{d\Omega_0}{dp} + (b_r^2 - b_i^{*2})\frac{d\Omega_1}{dp}\right] \cosh b_r c \sinh b_i^* c$$

$$+ \Big[3\left\langle -2b_r b_i^*\left(\frac{db_r}{dp}\right)^* + b_r^2 \frac{db_i}{dp} + b_i^{*2}\frac{db_i}{dp}\right\rangle \Omega_0^*$$

$$+ \left\langle 2b_r b_i^*\left(\frac{db_r}{dp}\right)^* - b_r^2\frac{db_i}{dp} + 3b_i^{*2}\frac{db_i}{dp}\right\rangle \Omega_1^* \Big] \sinh b_r c \cosh b_i^* c$$

$$+ b_i^*\left[(3b_r^2 + b_i^{*2})\frac{d\Omega_0}{dp} - (b_r^2 - b_i^{*2})\frac{d\Omega_1}{dp}\right] \sinh b_r c \cosh b_i^* c$$

$$+ c b_r\left[-(b_r^2 + 3b_i^{*2})\Omega_0^* + (b_r^2 - b_i^{*2})\Omega_1^*\right] \cdot$$

$$\cdot \left[-\sinh b_r c \sinh b_i^* c \left(\frac{db_r}{dp}\right)^* + \cosh b_r c \cosh b_i^* c \frac{db_i}{dp}\right]$$

$$+ c b_i^*\left[-(3b_r^2 + b_i^{*2})\Omega_0^* + (b_r^2 - b_i^{*2})\Omega_1^*\right] \cdot$$

$$\cdot \left[\cosh b_r c \cosh b_i^* c \left(\frac{db_r}{dp}\right)^* - \sinh b_r c \sinh b_i^* c \frac{db_i}{dp}\right] \Big\}$$

Case 3 -

$$b_r = \alpha + i\beta$$

$$b_i = \beta + i\alpha$$

$$\Omega_s = i\Omega_s^*$$

$$\Omega_s^* = \sinh^2\alpha(a-c+\epsilon_1) + \sin^2\beta(a-c+\epsilon_1) - [\sinh^2\alpha(a-c-\epsilon_1) + \sin^2\beta(a-c-\epsilon_1)]$$

$$\Omega_c = \sinh^2\alpha(a-c+\epsilon_1) - \sin^2\beta(a-c+\epsilon_1) - [\sinh^2\alpha(a-c-\epsilon_1) - \sin^2\beta(a-c-\epsilon_1)]$$

$$\Omega_o = i\Omega_o^*$$

$$\Omega_o^* = \alpha(\Omega_s^* - \Omega_c)\sinh 2\alpha a - \beta(\Omega_s^* + \Omega_c)\sin 2\beta a$$

$$\Omega_1 = \beta(-\Omega_s^* + \Omega_c)\sinh 2\alpha a - \alpha(\Omega_s^* + \Omega_c)\sin 2\beta a$$

$$\xi_m = \frac{1}{4m\alpha^2\beta^2(\alpha^2+\beta^2)^2}\sin\frac{\pi m\epsilon_2}{b}\sin\frac{\pi m d}{b}\sin\frac{\pi m y}{b}\cdot$$

$$\cdot\left\{\frac{1}{\sinh 2\alpha a\sin 2\beta a}\cdot\right.$$

$$\cdot\Big\langle\alpha\beta\Omega_o^*(\alpha\sinh 2\alpha x + \beta\sin 2\beta x)$$

$$-[\alpha\beta\Omega_1 + (-\alpha^2+\beta^2)\Omega_o^*](\alpha\sin 2\beta x - \beta\sinh 2\alpha x)\Big\rangle$$

$$-2(\alpha^2+\beta^2)\Big\langle[\alpha^2\sin^2\beta(x-c+\epsilon_1) - \beta^2\sinh^2\alpha(x-c+\epsilon_1)]U(x-c+\epsilon_1)$$

$$\left.-[\alpha^2\sin^2\beta(x-c-\epsilon_1) - \beta^2\sinh^2\alpha(x-c-\epsilon_1)]U(x-c-\epsilon_1)\Big\rangle\right\}$$

$$\lambda_m = \frac{1}{16m\left[\alpha\beta(\alpha^2+\beta^2)\right]^2} \sin\frac{\pi m \epsilon_2}{b} \sin^2\frac{\pi m d}{b}$$

$$\mu_m = -\frac{4}{\sinh 2\alpha a \sin 2\beta a} \cdot$$

$$\cdot\Big\{(\alpha \sin 2\beta c - \beta \sinh 2\alpha c)\left[\alpha\beta \mathcal{R}_1 - (\alpha^2-\beta^2)\mathcal{R}_0^{*}\right]$$

$$-\alpha\beta\mathcal{R}_0^{*}(\alpha \sinh 2\alpha c + \beta \sin 2\beta c)\Big\}$$

$$\nu_m = -8(\alpha^2+\beta^2)\left\{\alpha^2 \sin^2\beta\epsilon_1 - \beta^2 \sinh^2\alpha\epsilon_1\right\}$$

$$C_m^{*} = \frac{1}{4(\alpha^2+\beta^2)} \cdot \frac{\rho h}{D_x}\left[\frac{\rho h}{D_x} p_y^2 - \frac{D_y}{D_x}\left(\frac{\pi m}{b}\right)^4\right]^{-1/2} p_y$$

$$\frac{dk_r}{dp} = C_m^{*}(\alpha - i\beta)$$

$$\frac{dk_i}{dp} = C_m^{*}(\beta - i\alpha)$$

$$\frac{d\mathcal{R}_s}{dp} = C_m^{*}\Big\{(a-c+\epsilon_1)\left[\alpha \sin 2\beta(a-c+\epsilon_1) + \beta \sinh 2\alpha(a-c+\epsilon_1)\right]$$

$$-(a-c-\epsilon_1)\left[\alpha \sin 2\beta(a-c-\epsilon_1) + \beta \sinh 2\alpha(a-c-\epsilon_1)\right]\Big\}$$

$$\frac{d\mathcal{R}_c}{dp} = i\left(\frac{d\mathcal{R}_c}{dp}\right)^{*}$$

$$\left(\frac{d\mathcal{R}_c}{dp}\right)^{*} = C_m^{*}\Big\{(a-c+\epsilon_1)\left[\alpha \sin 2\beta(a-c+\epsilon_1) - \beta \sinh 2\alpha(a-c+\epsilon_1)\right]$$

$$-(a-c-\epsilon_1)\left[\alpha \sin 2\beta(a-c-\epsilon_1) - \beta \sinh 2\alpha(a-c-\epsilon_1)\right]\Big\}$$

$$\frac{d\mathcal{R}_0}{dp} = C_m^{*}\Big\{\left[\mathcal{R}_s^{*} - \mathcal{R}_c\right]\beta \sinh 2\alpha a - \left[\mathcal{R}_s^{*} + \mathcal{R}_c\right]\alpha \sin 2\beta a\Big\}$$

$$+\left[\frac{d\mathcal{R}_s}{dp} + \left(\frac{d\mathcal{R}_c}{dp}\right)^{*}\right]\alpha \sinh 2\alpha a - \left[\frac{d\mathcal{R}_s}{dp} - \left(\frac{d\mathcal{R}_c}{dp}\right)^{*}\right]\beta \sin 2\beta a$$

$$-4aC_m^{*}\alpha\beta\Big\{\mathcal{R}_c(1 + \sinh^2\alpha a - \sin^2\beta a)$$

$$-\mathcal{R}_s^{*}(\sinh^2\alpha a + \sin^2\beta a)\Big\}$$

$$\frac{d\Omega_1}{dp} = i\left(\frac{d\Omega_1}{dp}\right)^*$$

$$\begin{aligned}\left(\frac{d\Omega_1}{dp}\right)^* = C_m^* &\left\{(\Omega_s^* - \Omega_c)\,\alpha \sinh 2\alpha a + (\Omega_s^* + \Omega_c)\,\beta \sin 2\beta a\right\} \\ &+ \left[\frac{d\Omega_s}{dp} - \left(\frac{d\Omega_c}{dp}\right)^*\right]\alpha \sin 2\beta a + \left[\frac{d\Omega_s}{dp} + \left(\frac{d\Omega_c}{dp}\right)^*\right]\beta \sinh 2\alpha a \\ &+ 2aC_m^*\left\{\left[(\alpha^2-\beta^2)\Omega_c + (\alpha^2+\beta^2)\Omega_s^*\right](1+\sinh^2\alpha a - \sin^2\beta a)\right. \\ &\qquad \left. - \left[(\alpha^2+\beta^2)\Omega_c + (\alpha^2-\beta^2)\Omega_s^*\right](\sinh^2\alpha a + \sin^2\beta a)\right\}\end{aligned}$$

$$\frac{d\lambda_m}{dp} = i\left(\frac{d\lambda_m}{dp}\right)^*$$

$$\left(\frac{d\lambda_m}{dp}\right)^* = \frac{C_m^*}{8m\left[\alpha\beta(\alpha^2+\beta^2)\right]^3}\cdot(\alpha^4+6\alpha^2\beta^2+\beta^4)\sin\frac{\pi m \epsilon_2}{b}\sin^2\frac{\pi m d}{b}$$

$$\frac{d\nu_m}{dp} = i\left(\frac{d\nu_m}{dp}\right)^*$$

$$\begin{aligned}\left(\frac{d\nu_m}{dp}\right)^* = 8C_m^* &\left\{2\alpha\beta\left[(\alpha^2-\beta^2)(\sinh^2\alpha\epsilon_1 + \sin^2\beta\epsilon_1)\right.\right. \\ &\qquad \left. - 2(\alpha^2+\beta^2)(\sinh^2\alpha\epsilon_1 - \sin^2\beta\epsilon_1)\right] \\ &\quad \left. + \epsilon_1(\alpha^2+\beta^2)(\alpha^3\sin 2\beta\epsilon_1 - \beta^3\sinh 2\alpha\epsilon_1)\right\}\end{aligned}$$

$$\frac{d\mu_m}{dp} = i\left(\frac{d\mu_m}{dp}\right)^*$$

$$\left(\frac{d\mu_m}{dp}\right)^* = -\frac{8a\,C_m^*}{\sinh^2 2\alpha a\,\sin^2 2\beta a}\cdot(\alpha\sinh 2\alpha a\cos 2\beta a + \beta\cosh 2\alpha a\sin 2\beta a)\cdot$$

$$\cdot\Big\{-(\alpha^3\sin 2\beta c + \beta^3\sinh 2\alpha c)\,\Omega_0^*$$

$$+\alpha\beta(\alpha\sin 2\beta c - \beta\sinh 2\alpha c)\,\Omega_1\Big\}$$

$$+\frac{4}{\sinh 2\alpha a\,\sin 2\beta a}\cdot$$

$$\cdot\Big\{-3C_m^*\alpha\beta(\alpha\sin 2\beta c + \beta\sinh 2\alpha c)\,\Omega_0^*$$

$$+C_m^*\left[\alpha(\alpha^2+2\beta^2)\sin 2\beta c - \beta(2\alpha^2+\beta^2)\sinh 2\alpha c\right]\Omega_1$$

$$-(\alpha^3\sin 2\beta c + \beta^3\sinh 2\alpha c)\frac{d\Omega_0}{dp}$$

$$-\alpha\beta(\alpha\sin 2\beta c - \beta\sinh 2\alpha c)\left(\frac{d\Omega_1}{dp}\right)^*$$

$$+2C_m^* c\Big\langle -\big[(\alpha^4+\beta^4)(1+\sinh^2\alpha c - \sin^2\beta c)$$

$$-(\alpha^4-\beta^4)(\sinh^2\alpha c + \sin^2\beta c)\big]\,\Omega_0^*$$

$$+\alpha\beta\big[(\alpha^2-\beta^2)(1+\sinh^2\alpha c - \sin^2\beta c)$$

$$-(\alpha^2+\beta^2)(\sinh^2\alpha c + \sin^2\beta c)\big]\,\Omega_1\Big\rangle\Big\}$$

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NAVSHIPRANDCEN, Bethesda, MD 20034 (Attn: Code 173.2, Mr. W.P. Cauch)	1
NAVSHIPRANDCEN, Annapolis, MD 21402 (Attn: Code 2870, Mr. H. Edelstein)	1
NOL, White Oak, MD 20910 (Attn: Mr. F.R. Barnet)	1
NRL, Washington, DC 20375 (Attn: Dr. I.Wolock)	1

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